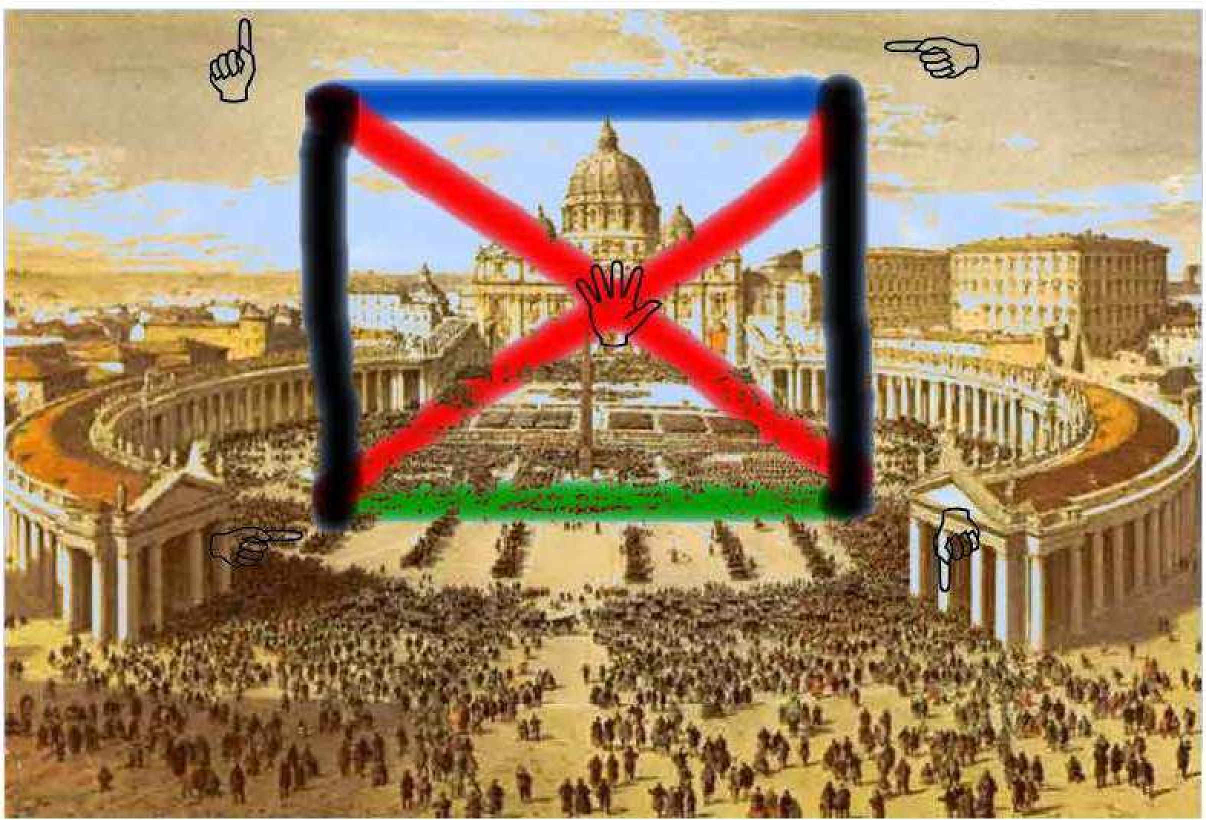
HANDBOOK OF THE WORLD CONGRESS ON THE SQUARE OF OPPOSITION IV



Pontifical Lateran University, Vatican
May 5-9, 2014

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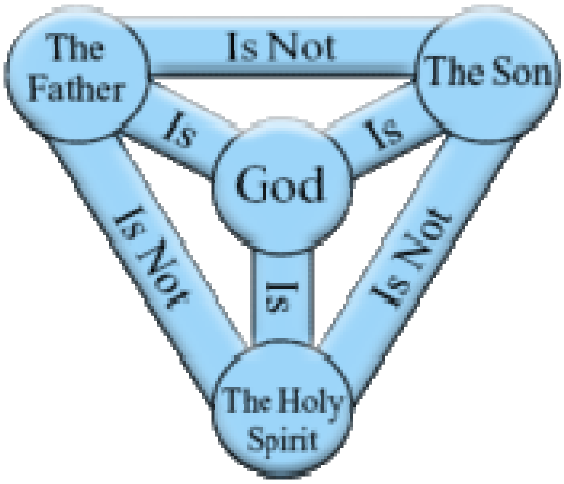
www.square-of-opposition.org

Contents

1. Fourth World Congress on the Square of Opposition

1.1. The Square : a Central Object for Thought

The square of opposition is a very famous theme related to Aristotelian logic dealing with the notions of opposition, negation, quantification and proposition. It has been continuously studied by people interested in logic, philosophy and Aristotle during two thousand years. Even Frege, one of the main founders of modern mathematical logic, uses it.
During the 20th century the interest for the square of opposition has been extended to many areas, cognitive science ultimately.

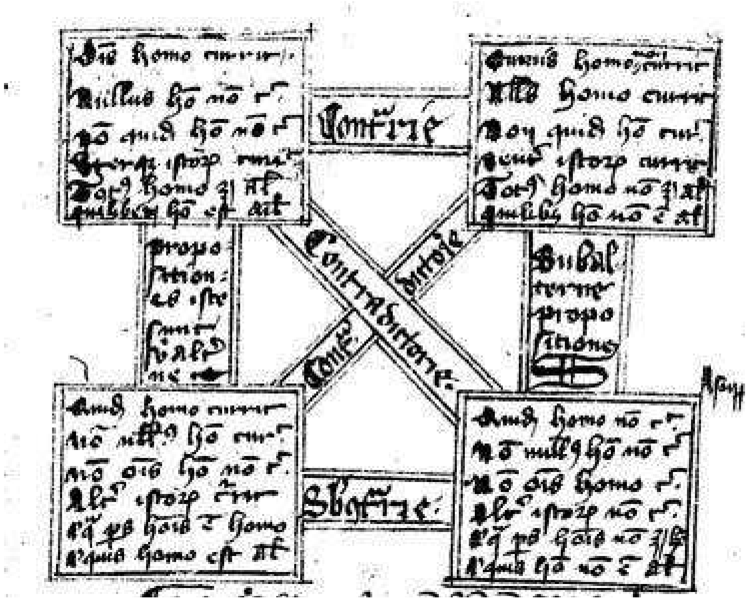


The trinity as a contrariety triangle

Some people have proposed to replace the square by a triangle, on the other hand the square has been generalized into more complex geometrical objects: hexagons, octagons and even polyhedra and multi-dimensional objects.

1.2. Aim of the Congress

This will be the 4th world congress organized about the square of opposition after very succesful previous events in Montreux, Switzerland 2007, Corté, Corsica 2010, Beirut, Lebanon, 2012. This is an interdisciplinary event gathering logicians, philosophers, mathematicians, semioticians, theologians, cognitivists, artists and computer scientists.



Buridan Square - Courtesy of Vatican

The meeting will end by a final round square table where subalterned people will express their various contrarieties, subcontrarieties and contradictions.

1.3. Scientific Committee

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2. Plenary Lectures

Gianfranco Basti

"Scientia una contrariorum": Paraconsistency, Induction, and Formal Ontology

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In this contribution, we offer an axiomatic presentation of the *Natural Realism* (NR) formal ontology, characterized by higher-order possible worlds. In its logic, a paraconsistent negation holds, so to justify the passage from a contradiction opposition to a contrariety opposition, when a new, more complex individual emerges from the entanglement of simpler ones. Before all, we present a synthetic formal treatment of the NR, as the proper formal ontology of the actual evolutionary cosmology. An issue for which some theoretical physicists and mathematicians tried to develop, at the foundation level, the theory of the "arboreal causal sets". NR ontology, is based, indeed, on the logic of the *converse implication,* <($p \leftarrow q$) (1101)>, and of its modal version, ($\neg\Diamond(q\wedge\neg p)$), as the logic of the *formal causality,* and hence as the logic of the *causal (ontological) entailment* ("a cause precedes *necessarily* its effect" (="causal necessity")), according to Aristotle's and Aquinas' suggestion. This is opposed to the logic of the *material implication* <($p \rightarrow q$) (1011)>, and of its modal version, ($\neg\Diamond(p\wedge\neg q)$), as the logic of the *logical necessity,* and hence, semantically, of the *logical entailment* ("a consequence *follows necessarily* from its premise"), according to C. I. Lewis theory of "strict implication". In this light, it becomes evident that W. V. O. Quine was right in criticizing the false Lewis pretension that its "strict implication" was a suitable logic for metaphysics, since it says nothing about the relation between the objects to which the statements, posed in the relation of implication, are referring. This criticism, on the contrary, can be satisfied if we take as logic of the metaphysical implication the completion of Lewis' theory of the logical entailment, with Aquinas' theory of the causal entailment. That is, "it is metaphysically (not logically) true that q formally follows from p *iff* (the referent of) p precedes causally (the referent of) q". Therefore, the Modal Logic (ML) of such causal processes cannot be **KT4** (or **S4**), but the proper ML of NR is **KD45,** or **secondary S5.** Its Quantified ML (QML) is a *possibilist* version (because of the axiom **D** instead of **T**) of the "objectual" **Q1R** system. In such a way, it is possible to formalize in NR an "arboreal" *unraveling* procedure of *causal constitution* (ancestor-descendants) – effectively a non-actualist, naturalist, version of R. Hayaki's "stipulation principle" - of *nested domains/sub-domains* of possible worlds, implementing a principle of "iterated modality" and of "stratified rigidity". In it, each level of the "unraveling" of equivalent domains (= new genus-species of individuals) has a **KD45** structure, and the whole system has a **nested KD45** structure, of growing complexity. It is evident, therefore, that in NR logic a paraconsistent negation holds, among the different levels and modes of *existence* characterizing it. Indeed, among its higher order possible world domains, affirmations and negations are not co-extensive.NR seems thus an optimal candidate as *formal ontology* of an evolutionary cosmology based on the Quantum Field Theory (QFT), as irreducible to the Quantum Mechanics (QM). In QFT ontology, the simpler beings (particles), existing in the past as individuals (e.g., the electrons existing as free particles in the hot plasma of our sun some billion years ago), exist "virtually" *now* inside a more complex, "actually" existing individual (i.e., they exist as quanta of electronic force fields trapped within an atom). What were mutually exclusive properties (in a *contradiction* opposition) of individuals (e.g., their opposite spins), now are complementary properties (in a *contrariety* max-min opposition) of one only "phase coherence domain" of force fields (i.e., the QFT justification of the quantum entanglement).

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Manuel Correia Machuca

The Didactic and Theoretical Expositions of the Square of Opposition in Aristotelian logic

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The Square of opposition is one of the best known and perhaps studied issues in the teaching of classical logic. What is less known however is the fact that the Square comes from the proto - exposition of categorical logic, which is a way to present a unified theory on what Aristotle and the first Peripatetics wrote and taught about categorical logic. The best textual representation of this proto - exposition is what is treated by Apuleius (II AD) and Boethius (VI AD). To speak of 'Square' is to simplify things, because neither Boethius nor Apuleius speak of a Square, even if they plot a square (when adding the subaltern relation which were not textually in Aristotle) and rather call it figure or diagram. What is less known yet is that the proto - exposition does not develop the Square relations in full extent. Thus, the internal relations of opposition and subalternation are not all that those that can be found and what have been found and traditionally taught has not been fully defined. However, this can be done through two classical properties of the categorical proposition, the quantity of the propositions and the quantity of terms, when accepting a set of axioms that previously allowed to extend categorical syllogistic by including the indefinite terms in the premises of syllogism. This simple move also allows both to extend the limits of categorical logic and to restore its theoretical unit, by distinguishing traditional and didactic exposition of the Square (and even of the entire categorical logic).

Lorenzo Magnani

Violence Hexagon. Moral Philosophy through Drawing

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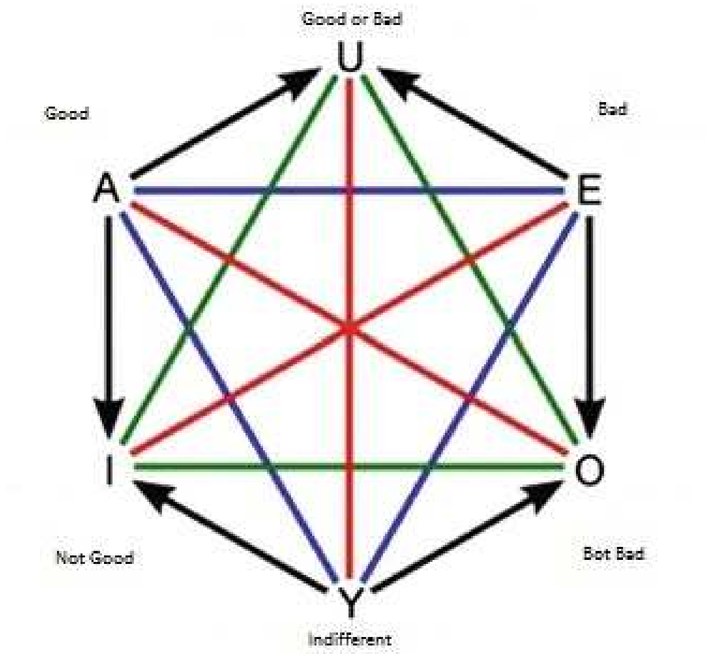
In this talk I will show why and how it is useful to exploit the hexagon of opposition to have a better and new understanding of the relationships between morality and violence and of fundamental axiological concepts. I will take advantage of the analysis provided in my book *Understanding Violence* (2011) to stress some aspects of the relationship between morality and violence, also reworking some ideas by Woods (2013), concerning the so-called



epistemic bubbles, to reach and describe my own concept of *moral bubbles*. The study aims at providing a simple theory of basic concepts of moral philosophy, which extracts and clarifies the strict relationship between morality and violence and more, for example the new philosophical concept of *overmorality*. I will also stress that this kind of hybrid diagrammatic reasoning is a remarkable example of *manipulative abduction* through drawing, in the spirit of Béziau's "conceptual structuralism".

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Rusty Jones

Bivalence and Contradictory Pairs in Aristotle's *De Interpretatione*

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Consider Aristotle's account of a statement: An affirmation states something of (*kata*) something; a negation states something from (*apo*) something. The one makes a claim about combination; the other makes a claim about separation. The principle of bivalence (PB) concerns statements: For every statement *p*, either *p* is true or *p* is false.

Now consider not single statements but contradictory pairs of statements. In a contradictory pair, one member statement (the affirmation) states something *of* something, while the other member statement (the negation) concerns the very same things as the affirmation, but states the former *from* the latter. The rule of contradictory pairs (RCP) concerns contradictory pairs of statements: For every contradictory pair (*p*,*q*), one of *p* and *q* is true, and the other is false.

Perhaps surprisingly, Aristotle denies both PB and RCP. In *De Interpretatione*, Aristotle discusses no fewer than three different sorts of statements and their associated contradictory pairs that serve as counterexamples to one or both of these principles.

My task in this lecture is to explicate Aristotle's account of bivalence and contradictory pairs. I will show why he formulates the principles as he does, and why he rejects them for certain classes of statements. Having done this, I will remark on the implications of this for understanding non-contradiction and the square of opposition. For those interested in some background to this lecture, please see:
https://www.academia.edu/2249267/Truth_and_Contradiction_in_Aristotles_De_Interpretatione_6-9

Bora İ Kumova

Symmetric Properties of the Syllogistic System Inherited from the Square of Opposition

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Although the logical square specifies the most simplest structure of quantifier relationships, by spanning a fully symmetric and self-complete logic, it was never successful in modelling realistic systems, since the existential quantifier is too powerful and the generalisation quantifier too strong. Furthermore, the latter is a special case of the former, making the square a logic with inclusive values [Moretti, A; 2012]. However, realistic systems use exclusive logical values, employ adaptive quantifiers, yet are self-complete. Categorical syllogisms are an immediate application of the logical square to logical inferencing with quantified propositions. The syllogistic system inherits all properties of the square, to all the four figures and to the well known 256 syllogistic moods, making the system impractical as a whole. Reductions of the whole syllogistic system to quantification-free, pure propositional rules of inference, like modus ponens or modus tollens, have proved to be the only practical

ones in engineering simple logical systems. However, such simplified inference rules were never successful in the engineering of more complex logical systems, not even their fuzzified generalisations. Since the syllogistic system encapsulates all possible inferences for given three objects, it represents a powerful inference system. Therefore, attempts are made to generalise the syllogistic system in various ways [Pereira-Fariña, M; et al; 2014] and use it as one complex inference method. With this objective in mind, we analyse the symmetric properties of the syllogistic system systematically, in order to propose a fuzzy syllogistic system that consists of fuzzy quantifiers, preserves symmetric structures, while being adaptive to application domains. As a by-product of the fuzzy syllogistic system and its properties, we suggest that in return the logical square may be generalised to a multigon.

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Wolfgang Lenzen

Leibniz's Logic and the Double Square of Opposition

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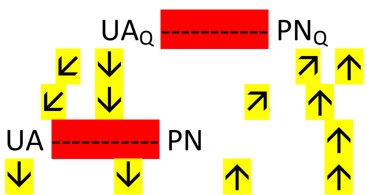
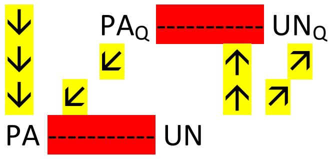
In this talk we will give an outline of Leibniz's logic (as reconstructed in more detail in Lenzen (2005)) and consider in particular his theory of the "Quantification of the Predicate" as developed in the paper "Mathesis rationis" of around 1700. While the traditional theory of the syllogism works with the four categorical forms 'Every S is P', 'Some S is P', 'No S is P', and 'Some S isn't P', Leibniz's "Quantification of the Predicate" yields in addition four unorthodox propositions 'Every S is every P', 'Some S is every P', 'Every S isn't some P', and 'Some S isn't some P' in the sense of the following formulas of first order logic:

UA_Q	$\forall x(Sx \rightarrow \forall y(Py \rightarrow y=x))$
PA_Q	$\exists x(Sx \wedge \forall y(Py \rightarrow y=x))$
UN_Q	$\forall x(Sx \rightarrow \exists y(Py \wedge y\neq x))$
PN_Q	$\exists x(Sx \wedge \exists y(Py \wedge y\neq x))$.

Since these unorthodox propositions satisfy the same logical laws of subalternation and opposition as the orthodox ones, they form another Square of Opposition. And since furthermore UA_Q entails its orthodox counterpart UA $=\forall x(Sx \rightarrow Px)$, both Squares can be joined together into the following "Cube of Opposition" where the red horizontal lines symbolize negations, while the yellow lines on the left and on the right hand side symbolize logical implications:





In the last part of the talk it will be shown that all these propositions can be formalized *within Leibniz's own logic*. E.g., UA_Q may be shown to be equivalent to requiring (1) that concept S contains concept P, $S \in P$, and (2) that both S and P are *individual concepts*. The latter condition itself may be defined in the framework of Leibniz's logic by requiring that S is *maximally consistent*: $I(S) \leftrightarrow \forall Y(S \in Y \leftrightarrow S \notin {\sim}Y)$ (where '~' symbolizes concept negation).

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Frederic Nef

Ontological square

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The ontological square is both an extension of the logical square of Boethius and an interpretation of the second chapter of Aristotle's Categories where a short list of categories was made. In contemporary metaphysics this ontological square has received a new attention. Jonathan Lowe wrote an important treatise on The Four Categories Ontology (Oxford University Press, 2006). The analytical ontology of objects, properties, kinds ans modes (tropes) is founded in that book on an aristotelician categorial ontology.

I will examine Jonathan Lowe's use of ontological square from the point of view of logical syntax and formal ontology. I will insist on the substitution of formal relations (caracterisation, instanciation) existing in the ontological square to logical relations existing in the logical square (contradiction, contrariety, subalternation).

This presentation will be an hommage to the memory of the great Aristotelian metaphysician Jonathan Lowe, who died very recently at the age of 62 in Durham.

Henri Prade

A Hexagonal View o Artificial Intelligence

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Knowledge representation and reasoning are key issues in artificial intelligence (AI). Robert Blanché has emphasized the importance of the square of oppositions and of its hexagonal

extension in the organization of conceptual structures at the time when AI was still in its infancy. However, these notions do not seem to have been considered in AI until recently, although their relevance for AI is genuine. In fact, the Aristotelian square can be completed into what may be called a cube of oppositions, by considering the introduction of a third negation. This does not just duplicate the square, since all the facets of the cube are meaningful. In particular, any binary relation induces such a cube of oppositions. Moreover, Piaget's group is at work inside the cube. The vertices of the cube can receive remarkable interpretations in different settings. In particular, one facet of the cube corresponds to the core of rough set theory, while basic formal concept analysis operators are found on another facet.
Each facet of the cube has a meaningful hexagonal extension. This cube, as well as squares and hexagons, can be encountered in many settings including classical and modal logics, possibility theory, formal concept analysis, rough set theory, argumentation theory (in terms of abstract attack relation, or when dealing with the forms of argumentative statements), generalized possibilistic logic, or logical proportions. Structure of oppositions may thus contribute to the foundations of a unified view of various frameworks for information processing in AI.

John Woods

How Globalization Makes Inconsistency Unrecognizable

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Properties, certainly many of the ones that interest logicians, come with what might be called ranges of application, whose members are those things to which the property in question could be intelligibly ascribed. (Provability can be intelligibly ascribed to statements but not, we may presume, to cabbages.) it is clear that some properties are valuable to theorists and others not. In large ranges of cases, the value a property has is lost if the set of its true instantiations exhausts its range of application. Provability and deducibility are such properties, as are sentence-validity and entailment. These are properties that partition their ranges in telling ways, hence, as we may say, are P-properties. They are also valuable for the conduct of any knowledge-seeking theory.

It has long been known that in classical environments, a single instance of a system's negation-inconsistency collapses the partition between correct and incorrect attributions of some of its P-properties, depriving them of their value, disabling them for gainful employment, drowning them in pathological over-instantiation, and dooming them to epistemic futility. The cause of the collapse is the classical equivalence between negation- and absolute inconsistency, under the provisions of the *ex falso quodlibet* theorem, provable in turn by the Lewis-Langford proof. *Ex falso* provides that a local inconsistency in a system *globalizes it* there. It also globalizes the cream of its P-properties, notably the four mentioned above.

Many attempts have been made to annul the globalization effect by discrediting the Lewis-Langford proof. Unsuccessful would be a charitable word for them, some a good deal more so than others. The strongest of these failed efforts arises from dialethic logic. The purpose of this paper is to demonstrate the failure of the dialethic option. As a kind of bonus we will come to see what Quine *actually* meant when he said that the dialethism option made the concept of negation unrecognizable.

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3. Abstracts of Contributors

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The Sum and the Product of Logical Hexagons

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The "logical square" or "square of opposition" is experiencing a renewal of interest, especially since the recent discovery by some authors (like Béziau, Moretti, Pellissier, Smessaert and Guitart) that its 1950 successor, the "logical hexagon" (co-discovered independently by Jacoby, Sesmat and Blanché), is part of a seemingly autonomous formal discipline, "oppositional geometry". Inside this emerging new field, which is even seen by some as a possible candidate for being, in some future, a new branch of mathematics of its own (comparable in that respect to knot theory or graph theory), some interesting invariants have been already brought to light. For instance, the square and the hexagon have been demonstrated to be just the first two terms of an infinite series of oppositional structures, the "*n*-oppositions", also called the "oppositional bi-simplexes of dimension *m*". In a similar way, other puzzling but nice oppositional structures, like Sauriol's 3D "logical tetrahexahedron" (1968), made of six nested logical hexagons, have been shown to belong to a second infinite series of regular oppositional structures, that of the "oppositional closures". "Matching results" allow mapping any member of the first to a member of the second series. Moreover, an infinite family of free oppositional structures (other than the previous), extending some graphs already used by people like A.N. Prior in modal logic, have been shown to be the "generators" of the previous two series of oppositional structures (the *n*-oppositions ad the *n*-closures). What is still missing, in order to start fulfilling the above prediction according to which oppositional geometry could be a new branch of mathematics (one dealing specifically with "oppositions") is, among others, a characterisation in terms of "category theory", given that the latter is the current "alphabet" of general mathematics. In this paper we point out a related problem: the absence so far, for oppositional geometry, of a "sum" and of a "product" operations. Focussing on the logical hexagon, which in some sense is one of the most important structures of oppositional geometry (for it is both very simple and highly powerful), we enquire the possibility of building formally and of interpreting philosophically for it such until now hypothetical operations. Thence we try to draw more general conclusions.

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Sidney Axinn

Toward The Logic of Ambivalence: the Cube of Opposition

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Several figures in the history of philosophy have made the assumption that the individual human is ambivalent. Heraclitus, St Augustine, and Kant are among them. The problem then becomes how to make an ordinary ambivalent person look rational. This paper provides **a cube of opposition** to accomplish just that purpose.[1]

For ambivalence to be displayed we must have at least one individual subject, one goal, and one pattern of opposition. The individual, A, either 1) accepts, or 2) rejects, or 3) neither accepts nor rejects an objective B. To simplify we will assume that the third case, 3) does not occur. Consider a traditional square of opposition in which one corner, #1, reads “A always rejects B,” the contrary corner, #2, reads ”A never rejects B,” the contradiction to #1, #3 reads “A sometimes does not reject B,” and the fourth corner, #4 reads “A sometimes rejects B.”

Now consider another traditional Square of Opposition in which the term, “reject” in our first Square is replaced by the term “accepts.” Then, we place each of these “accept” and “reject” squares on opposite sides of a cube. We can interpret this as a “Cube of Psychological Opposition.” Each line presents one kind of ambivalence: ambivalence consisting in vacillation between the positions at each end of that line, each corner. For the cube there will be twelve kinds of ambivalence, as in the 12 pairs of patterns at the ends of each line. Plus, The six faces on the cube can each have a pair of diagonals adding 12more. Plus, There are four interior diagonals, adding 4 more, for a total of 28 lines connecting opposite corners of the cube. Each of these lines can be taken to represent a variety of ambivalence between the opposite positions at the end of the line.

The paper distinguishes ambivalence between contradictories, contraries, subcontraries, subalterns, and equivalences. Some ambivalences can be maintained by slowly vacillating between alternatives, others by rapid vacillation. Some may be called “healthy, “ and some not.

The remainder of the paper consists in analyzing the Historical Relations involving ambivalence, The Dualist model of Human nature, and Ambivalence and Honesty.

François Beets

From the Square of Oppositions to the Coincidence of Opposites

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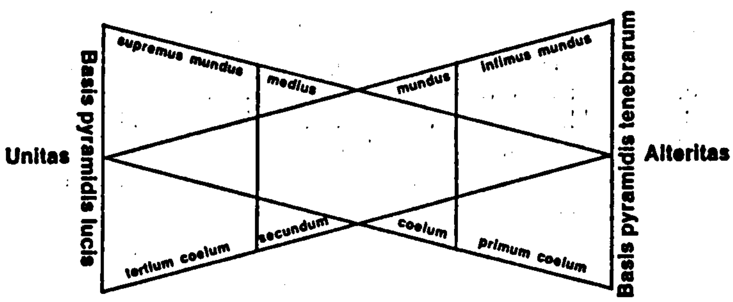
[1] Some of this material is taken from the author’s *The Logic of Hope: Extensions of Kant’s View of* Religion (Amsterdam: Rodopi Pubishers), 1994.

In the middle of the fifteenth century Nicholas of Cusa (1401-1464) criticized the Aristotelians for insisting on the principle of noncontradiction and refusing to admit the compatibility of contradictories in reality. For him the weakness of human reason is evident because of its primary rule, the principle of noncontradiction, which states that contradictories cannot be simultaneously true of the same object. This principle of noncontradiction rules the relations between categorical propositions as they were studied by Aristotle, categorical propositions, which have later been schemed in a square of oppositions known to any student in logic. But for Cusa there is, in reality, a “coincidence of opposite”, especially in the infinite God. Man is wise only if he is aware of the limits of the mind in knowing the truth. Knowledge is at best conjecture (*coniectura*). We definitely do not know the truth of any categorical proposition. Fortunately man has a power of knowing superior to reason: the *intellectus* or intuition, by which we rise above the principle of noncontradiction and see the unity and coincidence of opposites in reality. As a well-trained mathematician and an imaginative geometer Nicholas proposed us, in his *De docte ignorantia* and in his *De coniecturis*, some marvelous schemes. I will particularly stress on the scheme “P” (“P” both for paradigmatic and pyramidal), a scheme which is in some way the negative of the square of opposition.

Scheme P



The aim of my paper will be to investigate the meaning of this scheme and its relations to the square of opposition.

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Rainhard Z. Bengez

On Deontic Hypercubes, Absolut, and Relatives Rights

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Deontic squares are a well-investigated subject in legal theory, ethics, and logic. In this article we are aiming at making things a little bit more complex by escalating normative language structures into a normative logical hypercube. Associated with this little bit more complex deontic structure is the combinatorial determination of scope of logical design. In our presentation and associated paper we want to provide, firstly, the deontic logical hypercube and its genesis from underlying logical and deontic logical squares; secondly, we will provide a first glance of the underling formal legal modal logic of combinatorial determination of deontic structures. Hint: These two works are based on Lothar Philipps' research in legal theory and legal informatics.

Juan Manuel Campos Benítez

How to pass from the Square to the Octagon of Opposition

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The octagons of opposition clearly appear in 14th-century authors, and the difference from previous logic texts is remarkable, beginning with the number of pages devoted to the subject. The square of opposition was studied by such13th-century medieval authors as Peter of Spain, Lambert of Auxerre, William of Sherwood and a pseudo-Thomas Aquinas (since his *De propositionibus modalibus* is of dubious origin). The first three authors show squares of opposition with the traditional sentences A, E, I and O.

Modal squares are also exhibited with "sentences" such as *necesse est esse*, *necesse est non esse*, *possibile est esse* and *possibile est non esse* which, strictly speaking, are not sentences but a kind of schema. The pseudo-Thomas offers mnemonic devices such as amabimus, edentuli, purpurea and iliace for sets of sentences with the modes "necessary", "contingent", "impossible" and "necessary". Each vowel expresses the presence or absence of negation: "a" indicates no negation at all, "e" internal negation, "i" external negation and finally "u" internal and external negation.

14th-century authors such as Jean Buridan and Albert of Saxony developed octagons which combine sentences for quantification and de re modality, i.e. quantified sentences containing inside one of these modes. Jean Buridan also discusses octagons for quantified predicate sentences and oblique sentences (i.e. sentences in the genitive case, such as "for every man some donkey runs"). These octagons show corners with nine different but equivalent sentences.

In this paper I want to trace some of the elements that allow the construction of octagons, particularly in William of Sherwood. Indeed, Sherwood's treatment of predicate quantified sentences, especially when he speaks of their equivalent forms, allows a glimpse of Buridan´s octagon for sentences of "unusual construction" (*de modo loquendi inconsueto*). He offers a pair of rules to find equivalences. In fact Sherwood presents examples that correspond to the first four sentences of Buridan´s octagon. He also indicates how to obtain a hexagon from the usual square. However, Sherwood´s hexagon differs from the hexagons proposed by Béziau because its "new" corners are inside the square and do not exhibit any connective. In his *De propositionibus modalibus*, The pseudo-Thomas hints how to build a *de re* modal octagon for quantified modal sentences. This indication, however brief, can provide a vision of Buridan´s modal octagon.

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Plasticity in the Square – from a philosophical model to neurocognitive applications

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The Square of opposition deals with opposition, negation, quantification and proposition. What would happen if we embody the Square and apply it into a specifically-structured training? To this aim, it was necessary to create a simple and absolutely defined frame of reference, which allows the training of attention and negation, keeping alive the ability to respond readily and in a precise way to the unexpected new command. We thus started from a philosophical vision of man and what he can become. With this philosophical vision, we designed a scientific experiment. The current talk will address the multidisciplinary examination and results related to the effects of whole-body training within the Square. We will discuss the importance of (a) stimulating attention and perception; (b) changing the processes of calculation within the Square with the aim of stimulating brain areas in new ways; (c) incorporating coordination of the left and right sides of the body, and their implications related to unity from a cognitive and philosophical perspective.

Related to this, the embodied cognition hypothesis suggests that the neural networks related to cognition are closely related to perception and action. It further argues that concepts arise from sensorimotor activation, which in turn forms the building blocks of creative thought, a crucial ability for daily life. Cognitive and education neuroscience supports the embodied cognition hypothesis (Cosmelli and Thompson, 2009; Damasio, 2000; Paoletti and Salvagio, 2011;Varela et al., 1991). In order to reveal the underlying mechanisms related to the Square, we examined the *Quadrato Motor Training (QMT)*. The QMT is a sensorimotor training paradigm, constructed as a Square in which the participant moves according to specific oral instructions (Figure 1). The QMT is aimed at increasing attention, coordination and creativity.

The current talk will discuss the cognitive effects of the QMT. These will be integrated with electrophysiological, structural and molecular investigations related to the Square. The results from recent studies demonstrate that QMT can enhance functional connectivity (FC), and increase creativity and reflectivity (Ben-Soussan et al., 2013; 2014). In line with previous results, change in frontal FC was significantly correlated with change in creativity. FC sculpting (Silberstein, 2006) will be discussed in regards to the idea of reshaping our behaviors, perception and cognition by training, in various populations (e.g. healthy and dyslexic populations). The talk will end with the theoretical, cognitive and perceptual implications, also in the context of higher states of consciousness.

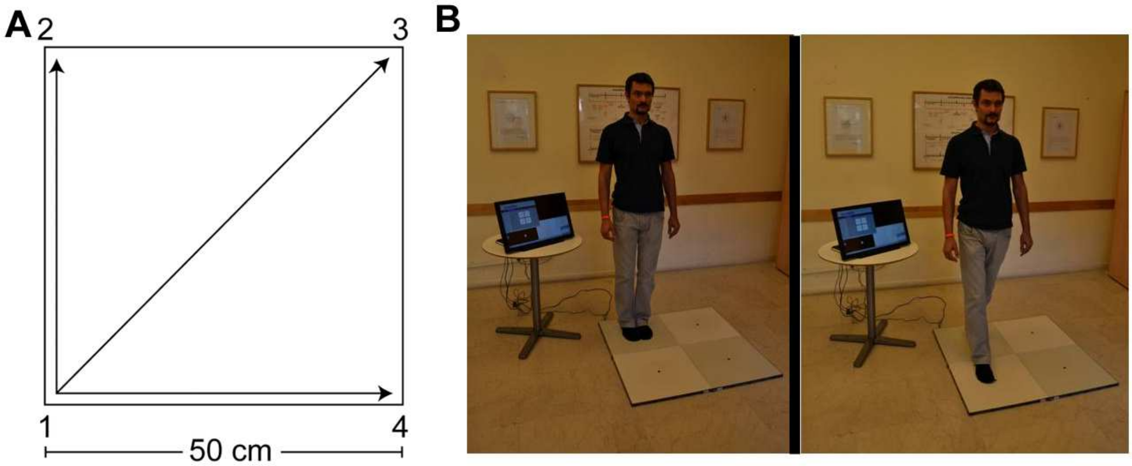

Figure 1. (A) The spatial layout of the training space. (B) Practice setup. The trainee listens to recorded instructions and takes a step towards the target point. (Figure 1 is adapted from Dotan Ben-Soussan et al., 2014).

The current talk will discuss the importance of bringing together different approaches to the understanding of the Square and the scientific value of sensorimotor training.

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Christoph Benzmüller and Bruno Woltzenlogel Paleo

Gödel's Proof of God's Existence

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Attempts to prove the existence (or non-existence) of God by means of abstract ontological arguments are an old tradition in philosophy and theology. Gödel's proof is a modern culmination of this tradition, following particularly the footsteps of Leibniz. Gödel defines God as a being who possesses all positive properties. He does not extensively discuss what positive properties are, but instead he states a few reasonable (but debatable) axioms that they should satisfy. Various slightly different versions of axioms and definitions have been considered by Gödel and by several philosophers who commented on his proof. We have analyzed Scott's version of Gödel's proof for the first-time with an unprecedented degree of detail and formality with the help of theorem provers; cf. https://github.com/FormalTheology/GoedelGod.

The following has been done (and in this order): (i) a detailed natural deduction proof; (ii) a formalization in TPTP THF syntax; (iii) an automatic verification of the consistency of the axioms and definitions with Nitpick; (iv) an automatic demonstration of the theorems with the provers LEO-II and Satallax; (v) a step-by-step formalization using the Coq proof assistant; (vi) a formalization using the Isabelle proof assistant, where the theorems (and some additional lemmata) have been automated with the tools Sledgehammer and Metis.

Gödel's proof is challenging to formalize and verify because it requires an expressive logical language with modal operators (*possibly* and *necessarily*) and with quantifiers for individuals and properties. Our computer-assisted formalizations rely on an embedding of the modal logic into higher-order logic with Henkin semantics. The formalization is thus essentially done in classical higher-order logic where quantified modal logic is emulated.

In our ongoing computer-assisted study of Gödel's proof our deduction tools have made some interesting observations, including: (a) The basic modal logic K is sufficient for proving the first three theorems (T1, Coro and T2) as outlined in Scott's notes. (b) For proving the final theorem (T3), logic KB is sufficient. (c) Gödel's original version of the proof, which omits conjunct $\phi(x)$ in the definition of essence, seems inconsistent.

This work attests the maturity of contemporary interactive and automated deduction tools for classical higher-order logic and demonstrates the elegance and practical relevance of the embedding-based approach. Most importantly, our work opens new perspectives for a computer-assisted theoretical philosophy. The critical discussion of the underlying concepts, definitions and axioms remains a human responsibility, but the computer can assist in building and checking rigorously correct logical arguments. In case of logico-philosophical disputes, the computer can check the disputing arguments and partially fulfill Leibniz' dictum: Calculemus — Let us calculate!

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Frode Bjørdal

The Cube of Opposition for Agentual Directives

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We distinguish between eight modalities formed from the formula forming formula operating operators O (it ought to be the case that ...) and B (the agent S brings it about that ...) as follows, where P as usual abbreviates ¬O¬ and (i,j,k) ∈ $\{0,1\}^3$ is the corresponding corner of the unit cube for intuition and visualization purposes.

OBq decorates (0,1,1)
OB¬q decorates (1,1,1)
O¬B¬q decorates (0,0,1)
O¬Bq decorates (1,0,1)
PBq decorates (0,1,0)
PB¬q decorates (1,1,0)
P¬B¬q decorates (0,0,0)
P¬Bq decorates (1,0,0)

The modality decorating a point (i,j,1) entails the modality decorating (i,j,0), as obligations entail permissions. The modality decorationg a point (i,1,k) entails the modality decorating (i,0,k) as bringing about that something happens entails that one is not bringing about the complementary state of affairs. OBq and OB¬q are contrary.

Long diagonals are between contradictory sentences, so we have a cube of oppositions.

Notice that this more fine grained apparatus isolates a peculiar inviolability directive, viz. when both the modality decorating (0,0,1) and the one decorating (1,0,1) are true. If a proposition is inviolable then also its negjunction is inviolable. Holy objects are inviolable in that they enter many propositions which are inviolable to many.

Just four modalities corresponding to a side of the cube are true, and exactly the four sides of the cube that do not contain both (0,1,1) and (1,1,1) correspond to a classically possible combination of modalities which express a directive.

Let q be adiaphoric iff q is permitted and also ¬q is permitted. T here are four distinct deontological directives possible for an agent with respect to a given q: (i) q is mandatory, (ii) q is forbidden, (iii) q is adiaphoric and (iv) q is inviolable. The author does not know whether or how the latter agentual directive has been discussed in the literature. Nevertheless, in certain cases where the agent is obligated to not interfere some propositions are correlated with the inviolability directive

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When the Square meets the Cross: Towards a generalized notion of entailment

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From time immemorial, the traditional Square of Oppositions has served its purpose of organizing the different notions of opposition from Aristotelian Logic into a coherent and suggestive pictorial form. The Square was repeatedly proven fit for survival particularly by remaining a powerful source of inspiration after its structure has resisted opposing forces of augmentation and of subtraction. Operating on our own idiosyncratic approach, for the sake of generality and symmetry, we will here simply assume a Square as a mother structure à la Bourbaki, designed to be concerned about the notions of Contradictoriness, Antonymicity and Duality.

Our main purpose in this contribution is to show how a Logical Structure may be extracted from a Square. To that effect we start from the Fregean-Tarskian notion of entailment that expresses judgments based on assertions and refutations, and then add a further dimension to it, by considering instead agents and their cognitive attitudes towards the informational quanta expressed by propositions conveyed by formal sentences about the world. In such generalized framework, that we call B-entailment, the usual two-place relation between a collection of sentences jointly taken as premisses and a collection of sentences taken as alternative conclusions is replaced by a (two-times-two)-place relation between suitable collections of sentences, giving rise to statements of the form: $\frac{A_{12}}{A_{11}} \Big| \frac{A_{22}}{A_{21}}$. If in the ordinary notion of entailment a judgment of the form $A_1 \vdash A_2$ fails to hold whenever there is a state-of-affairs in which the sentences in A_1 are simultaneously asserted while the sentences in A_2 are simultaneously refuted, according to the canonical interpretation of B-entailment, the statement $\frac{A_{12}}{A_{11}} \Big| \frac{A_{22}}{A_{21}}$ is said to fail according to some judgmental agent just in case this agent can simultaneously see reasons to accept the sentences in A_{11}, reasons to reject the sentences in A_{21}, reasons not to reject the sentences in A_{12}, and reasons not to accept the sentences in A_{22}.

After showing that for all practical matters the concept of B-entailment behaves precisely as one would expect from a generalization of the concept of logical consequence, we will also show how each of the characteristic notions of Opposition from the underlying Square are reflected into rotational movements that may be embodied into the B-statements, as seen from the viewpoint of Universal Logic. At last, we will also discuss an otherwise unsuspected connection between B-entailment and bilattice-based logical reasoning.

Walter Carnielli

Groups, not Squares: unveiling a fetish

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I argue that the celebrated Square of Opposition is just a manifestation of a much deeper relationship on duality, complementarity, opposition and quaternality expressed by algebraic means, and that any serious attempt to make sense of squares and cubes of opposition must take into account the theory of symmetry groups. An abstract account of

duality is discussed in [HG98], where it is suggested that there is a group of order four acting in the set of propositions (this is better appreciated by expressing propositions in polynomial format, as in [Car05]), not a group of order two, as it is commonly thought. This group is precisely the famous Klein 4-group, and therefore simple duality has no place in the deep relationship between algebra and logic: what holds is quaternality, as already remarked in [Got53]. This is what lies behind De Morgan laws and behind many results of universal Boolean algebra, geometry, topology, and several other areas.
I show that the algebraic structure behind Boolean groups B_n (a Boolean group is a finite abelian group in which every element different than the inverse *i* has order two) is what makes them relevant for understanding symmetries: as each B_n is a subgroup of B_{n+1}, they form increasingly complex structures, and any symbolic system may be regarded as a subsystem of a larger one. For instance, each face of the cube Q_3 is Q_2 so the “cube of oppositions” contains six “squares of oppositions”, and so on, expressing combinatorial manifolds of complex binary oppositions. In 1936 D. König, the graph-theorist who proposed the famous König’s Lemma, conjectured that every finite group is the group of symmetries of a finite (undirected) graph, a result proved by R. Frucht in 1939. Frucht’s theorem essentially says that for any finite group, there is a graph G such that the group of authormorphisms of G is isomorphic to the given group. In particular, this holds for any group of symmetries, so there will be infinitely many abstract forms of ‘squares of symmetries’ to play with. Recognizing that our understanding in many things has evolved since the Aristotelian doctrine of square and that combinatorial complexity is what explains some intuitions in linguistic, logic, set theory, category theory, topology, geometry, philosophy and anthropology would liberate people from the squared fetish.

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Massimiliano Carrara and Daniele Chiffi

A logical framework for hypotheses and their opposition relations

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Consider two hypotheses: what does it mean that the first one contradicts the other? Or that two hypotheses are one the contrary of the other? Or, again, what does it mean to say that one hypothesis is the subaltern of the other? Generally speaking: how to characterize relations of opposition among hypotheses?
Aim of the paper is to sketch a logical framework where a solution to the above questions is given: A logical square of hypotheses is formulated and relations of oppositions among them are scrutinized.

Our starting point will be Dalla Pozza and Garola logic for pragmatics, specifically the pragmatic interpretation of intuitionistic logic (ILP) (Dalla Pozza & Garola 1995) where sentences and proofs formalize assertions and their justifications. We sketch an extension of the pragmatic approach using some insights taken from a pragmatic logic for hypotheses (Bellin 2010). In such logic there is a way to give reason of the duality between assertions and hypotheses. Two kinds of negations, the assertive and the hypothetical ones, represent such duality.

We develop our goal in the following way.

First, we propose to conceive assertions as verified sentences, specifying how justification works in cases like *being justified to assert* or *being unjustified to assert*. We introduce a notion of *pragmatic contradiction* for justified and unjustified assertions. A pragmatic square of opposition for assertions is formulated where the relations of oppositions are characterized.

Second, using the basic ideas of hypotheses as a primitive illocutory force justified by means of a *scintilla of evidence,* by means of the logical tools developed in the above-mentioned logic of hypotheses (Bellin 2010), a square of opposition for hypotheses is formulated and compared with the square of opposition for assertions.

Finally, we propose a translation of the two given pragmatic squares for assertions and hypotheses into their modal counterparts of classical S4.

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Ferdinando Cavaliere

Iconic and Dynamic Models to represent 'distinctive' predicates: the Octagonal Prism and the Complex Tetrahedron of Opposition

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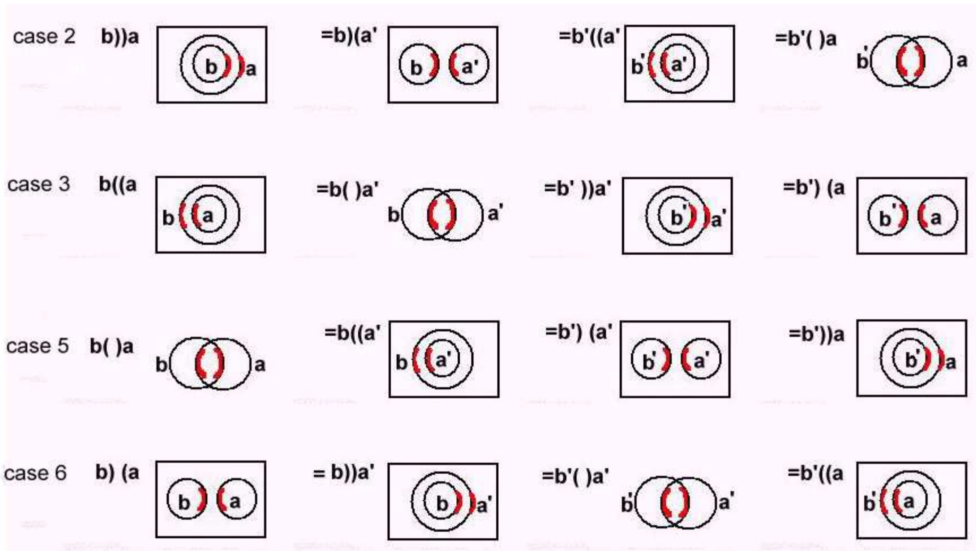
The predications of the Blanché Hexagon, enriched with converses and negative terms (ba - ab - b'a - ab' etc), can be integrated into the "Octagonal Prism" of Opposition, an exhaustive model drawn from the Distinctive Predicate Calculus (Cavaliere F., 2012). In its "compound" development, the Distinctive Predicate Calculus (DPC) presents seven base expressions with existential presuppositions. Here we present a version inspired on the one hand by the notational system of A. De Morgan (1847) and on the other by the relations of J. D. Gergonne (1816). This way 'distinctive' predicates and immediate inference rules are easily translated into diagrams (see examples in the figure below).



The seven predicates, added to the nine cases without existential presupposition, are exhaustive and geometrically organizable in a complex Tetrahedron of Opposition, consisting of four “actual” vertices plus twelve "virtual" ones. This model has the following ‘dynamic’ features: 1) the existential presupposition disappears from the barycentre, moving to the outer space; 2) each transition from a ‘predicative’ vertex to a contiguous one corresponds to the subtraction or addition of a single diagrammatic logical subset or sector; 3) the Tetrahedron has a substructure, the "Double Diamond", formed by the seven cases. It is a spatial itinerary, semantically interpretable in terms of concepts that are gradually opposite (none-weak-strong-total opposition), or gradually similar, if we reverse the order of the degrees (total-strong-weak-no similarity). This ‘scalar’ substructure can play an important role in culture. In a variety of theoretical and applied disciplines (semantic-web, library science, taxonomy, rhetoric, etc.) some concepts that are not definable in terms of classical logic become logically transparent by the pattern of the seven predicates of DPC.

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Saloua Chatti

Al Fārābī on contrary practical concepts

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In his *Kitāb at-Taḥlīl*, Al Fārābī presents an analysis of the propositions whose subjects and predicates are contrary concepts. He considers moral and practical concepts and raises the following problem: Which propositions of that kind are incompatible with each other, either for everyone or for some people? And which ones can be admitted together by the same person, and compatible with the same behavior or the same opinion? To answer these questions, he gives four examples of propositions involving contrary concepts of that kind and examines them systematically by considering all the possible combinations of concepts. In each case, he takes two contrary subjects and two other contrary predicates. The concepts involved are justice and injustice, good and evil, friend and enemy, harmful and kind, pleasure and pain, life and death. One of the examples contains also a deontic operator which is 'ought to'. All the combinations are presented in several tables, where each table contains 6 lines, and each line relates two opposed propositions. The six lines could correspond to the usual relations of the square, but Al Fārābī shows that the relations of the square do not all hold for these propositions, and that each example is different from the other ones. While the first example contains two equivalent propositions and several incompatible ones, the other ones admit several and various interpretations and depend on previous philosophical or common theses. Some couples of propositions involved in these examples are considered as subcontrary by some people and as contrary by other ones. They are thus fluctuant and not objectively determined.

In this talk, I will analyze these tables and the oppositions between the given propositions. I will show that Al Fārābī's analysis, although based on systematic combinations, remains incomplete in its characterization of the relations between the propositions. For instance, Al Fārābī mentions only one deontic operator ("ought to"); so one has to add the other deontic operators to provide a fuller account of the relations. This addition, however, produces different relations between the propositions depending on whether the concepts are contrary or contradictory. The analysis of the other examples requires the use of quantifiers and epistemic operators.

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Boris Chendov

Square of opposition and its enlargement in the dyadic modal logic

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1. The construction of various types of non-classical logical systems during 20th century raises the question about construction in the framework of some of them **(1)** of such kind of logical squares of opposition (sq. of op.), which have to be analogical to the classical sq. of op. formulated for Aristotelian syllogistic, more precisely speaking for relations between propositions of the type **A, I, E, O,** as well as **(2)** of their enlargement. The present paper is devoted **(1*)** to construction of a sq. of op. in the framework of the Dyadic Modal Logic (**DML**), the syntax and semantics of which was exposed in [1] and [3], as well as **(2*)** to its enlargement.

It is worthy of mentioning that this **DML** is a part of a more general as well as more abstract Dyadic Determination Logic revealing namely its modal (i.e. conserning the concepts of necessity, possibility and the like) aspect together with the other its part – Dyadic Prescriptive Logic (**DPrsL**), including the Dyadic Deontic Logic and the Logical System of Problems (of Questions). Accordingly the results about sq. of op. and its enlargement formulated in the present paper can be in principle reinterpreted in the feald of **DPrsL**.

2. The systematic construction of various dyadic modal concepts in the language of **DML** is realised in four levels. Correspondingly the problem **(1)** of construction of sq. of op. in **DML** which have to be analogical to the sq. of op. formulated for Aristotelian syllogistic, as well as **(2)** of its enlargement, obtain a peculiar form for every one of these four levels.

3. For dyadic modal concepts of the level 2 the following two squares of opposition takes place:

```
         x¯D¯y--------------x¯Dy                     xNy-------------------xLy
           |                  |                        |                   |
 First     |                  |          Second        |                   |
sq. of op. |                  |          sq. of op.    |                   |
          xDy-----------------xD¯y                    xDy----------------xD¯y
```

For the first sq. of op. "**D**" means D-possibility, "**¯D**" - D-impossibility, "**¯y**" – negation of **y**, "**¯D¯**" – D-necessity. The transition from the sq. of op. formulated for Aristotelian syllogistic to the first sq. of op. formulated above for the **DML** is realised on the grounds of following correspondences:

(1) Aab ÷ x¯D¯y , **(2) Eab ÷ x¯Dy**, **(3) Iab ÷ xDy**, **(4) Oab ÷ xD¯y**, provided **a ÷ x**, **b ÷ y**, where the sign "÷" means "correspondence" (cf. [2, p. 45]).

The second sq. of op. takes place for the system of **DML** obtained from the one exposed in [2] by means of a slight modification: the condition (respectively, the axiom) 6 (cf. [2, p. 44]) is omitted, so owing to this fact the barren sets (i.e. such kind of sets **x** for which **(x¯Dy & x¯D¯y)** is valid) are allowed. In this case "**N**" means N-necessity, "**L**" **–** L-impossibility, "**D**" – D-possibility, and "**¯D**" - D-impossibility; by definition:

(1) xNy ↔ (**xDy & x¯D ¯y**), **(2) xLy ↔** (**x¯Dy & x D ¯y**),

where "**α↔β**" means "**α→β & β→α**", **α** and **β** are propositions as well as "**→**" means implication in the sense of Classical Propositional Logic.

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Representing Existential Import With Quadrants

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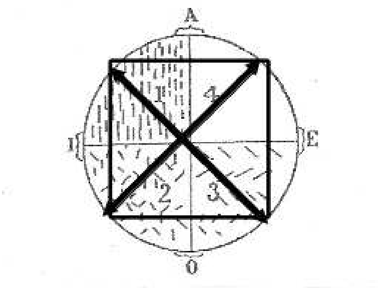
Boolean algebraic logic imposes a new condition on classes: they must not be null if we want to keep the traditional square of opposition. Existential import states that particular propositions must imply the existence of their terms and universal propositions must not, for they state what Peirce calls a "leading principle." If we want to use the algebraic notation with no restriction, there are no longer contraries, subcontraries and subalterns, and the only opposition left is contradiction. It leads to what Peirce calls "the collapse of the time-honored jingle about opposition."

It is striking that such an important matter as existential import did not find some place on the very spatial representation of opposition. Peirce did try to represent the existential import for particulars on a diagram, namely a circle of opposition, each quadrant of which shows a relationship between subject and predicate. The surfaces being filled in introduces a kind of third dimension. Eight propositions obtain, and four "possible universes." Fig. 2 shows that the "new quadrant" of opposition is akin to tables of truth like Fig. 3.



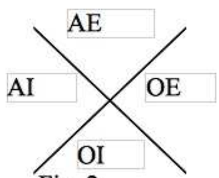


Fig. 2

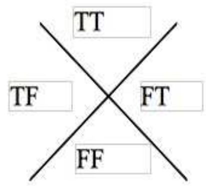


Fig. 3

Thanks to a π/4 rotation, the circle maps the square, revealing the potentialities of the square for representing existence. I address the question: is there a possible transition between such a new account of the square of opposition and diagrammatic logic? I then try to extend the "new square" or circle of opposition to propositions in "two dimensions," a phrase coined by Mitchell to describe not only existence in space but also through time.

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Kraszewski's Syllogistic

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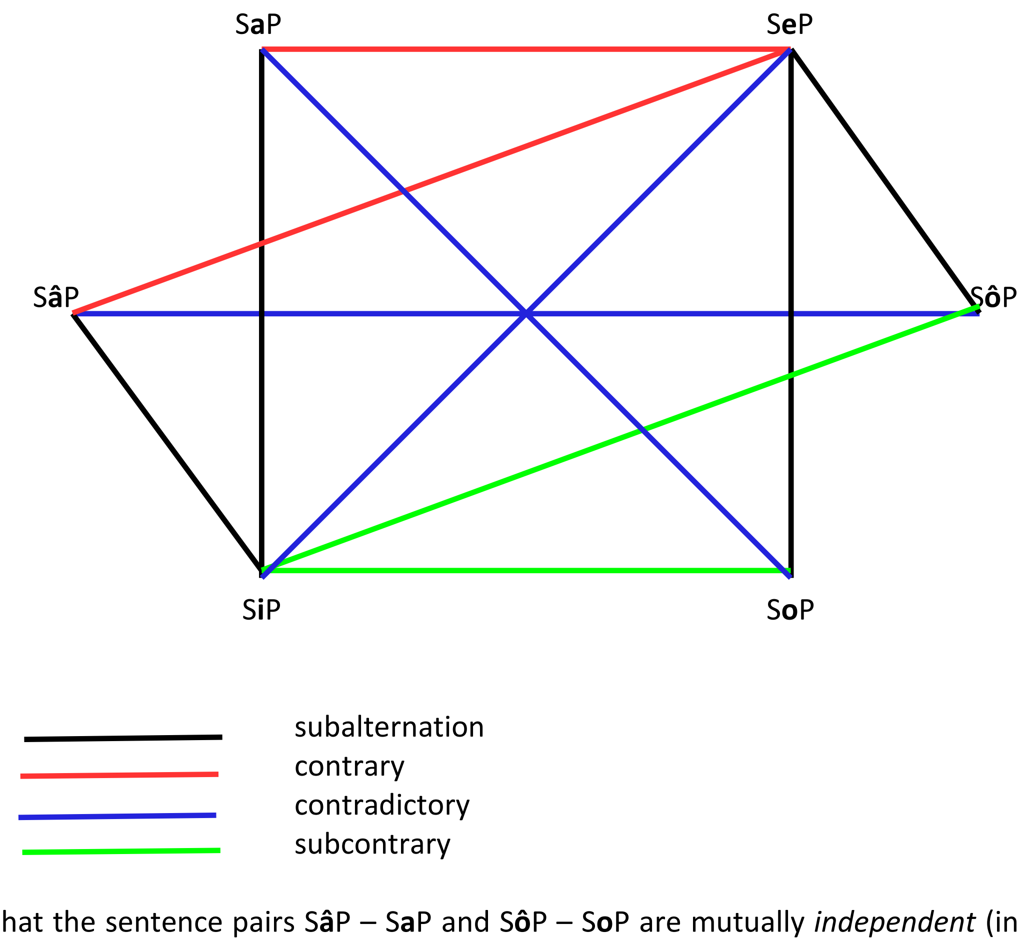
It is well-known that categorical sentences are divided into four fundamental classes. In 1956, Zdzisław Kraszewski published his paper on the non-Aristotelian theory of the syllogism. Kraszewski points out some problems with the semantic interpretation of the categorical sentences used in the premises of Aristotelian syllogisms. He notices that there are two additional classes of the categorical sentences which are not explicitly stated, but used implicitly: the conversion of categorical sentences. Since we have only two simple conversions (plus conversion by limitation) there is some *lopsidedness* between the sentences.

A natural counterweight to that phenomenon is, according to Kraszewski, to enrich the class of the categorical sentences with two more propositions: *Only S is P* (or equivalently: *Some S is all P*, S**â**P in symbols) and *Not only S is P* (or equivalently: *Not S alone is P*, S**ô**P in symbols).[2]

It results in obtaining the following *figure* of opposition:



Note that the sentence pairs S**â**P – S**a**P and S**ô**P – S**o**P are mutually *independent* (in a sense that there is not any relationship between them).

The aim of my talk is to present Kraszewski's syllogistic: philosophical background, main ideas and resulting formalism.

[2] Cf. Keynes (1906), *Chap. VII, §§ 146-148.*

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Two Kinds of Incompatibility in Law

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1. The deontic square of opposition defines three relationships of incompatibility between norms (antinomies). It is well-known that one of these relationships holds between contraries (*first relationship*: "*p* is obligatory" and "*p* is forbidden") and the other two hold between contradictiories (*second relationship*: "*p* is obligatory" and "*p* is facultative"; *third relationship*: "*p* is forbidden" and "*p* is permitted").

2. In the world of *Sein* (in descriptive language) two descriptive propositions are incompatible when they *cannot* be *both true*. However, in the world of *Sollen* (in prescriptive language) what does incompatibility between norms (normative incompatibility) consist of?

Some philosophers have maintained that two incompatible norms *cannot* be *both valid*; others have maintained that two incompatible norms *cannot* be *both fulfilled*.

3. I criticize both answers and believe that the answer cannot be the same for all the three relationships of incompatibility. The deontic square of opposition throws light indeed on this problem, since it allows us to distinguish between two kinds of normative incompatibility.

In the first relationship (contrariety) the incompatibility between norms lies at the level of the *agent*: it is impossible for the agent *to fulfil* both norms (*proheretic incompatibility*).

In the second and third relationship (contradiction), the incompatibility between norms lies at the level of the *judge*: it is impossible for the judge *to apply* both norms (*dikastic incompatibility*).

I discuss some examples of *proheretic* and *dikastic incompatibility* in order to make these concepts clearer.

Duilio D'Alfonso

Typing duality relations

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It is well-known that, given an operator *o*, its dual operator, generated by means of the application of the outer negation to the inner negation, is the "strong" version, provided that *o* is the "weak" version, and vice-versa. Let ∇be the weak version of an operator. The dual Δ, representing its strengthening, is defined as follows:

$$\nabla = \neg\, \Delta\, \neg$$

We can easily express the logical asymmetry between the weak and the strong version of an operator by considering that the following entailments hold:

$$\Delta \Rightarrow \nabla \text{ but } \nabla \not\Rightarrow \Delta.$$

So, the dual generation is a "necessitation" operation, when applied to a weak operator and, conversely, a "possibilization", when applied to a strong operator. Can we find reasons for choosing the weak, rather than the strong, version, as primitive of a logical system? Let consider the following list of well-known weak/strong pairs (Boolean connectives, standard quantifiers and modal, temporal (for future and past) and epistemic operators):

weak	strong
$\vee$	$\wedge$
$\exists$	$\forall$
$\Diamond$	$\Box$
F	*G*
P	*H*
Ḱ	*K*

The choose of the undefined operator seems to be, mainly, a matter of convenience, given a particular theoretical context. Namely, this seems to be true for Boolean connectives and quantifiers. For what concerns modal, temporal and epistemic operators, pragmatic issues, related to their representation in natural language, seem to become relevant. For instance, while the truth in a future instant (*F*), with respect to the time of utterance, is expressed, in many languages, by the future tense, the truth for ever in the future (*G*) is to be paraphrased (and similarly for the past). This could explain the preference accorded to the weak operators *F* and *P*, as primitive, in constructing out systems of temporal logic. A similar argumentation holds for the epistemic operator. *K* is preferred as it means: "an agent knows -", rather than *Ḱ* that means: "an agent cannot exclude -". Note that here it is the strong version to be selected as the undefined operator of the system. In the above list all cases of logical "asymmetry" between duals are presented. But also the "symmetric" case is to be contemplated. Consider the adverbial pair
still/*already*. We can define *already* as the dual of *still*:

already = ¬*still*¬

Neither *already* implies *still* nor *still* implies *already*, therefore this is what we can call a "weak symmetry". Since also cases of "strong symmetry" are known, when the duals imply each other (ending up to be equivalent, such as some natural language determiners), the dual generation is to be thought as a syntactic, and not semantic, process. Semantic aspects of operators, that are in some sense "idiosyncratic", are then responsible of the logical properties of the (a)symmetries here classified. In this paper I will argue for this claim, trying to draw some general implications, at the interface between logic and natural language.

Yusuf Dasdemir

Avicenna on the Opposition of Conditional Propositions

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Conditional propositions, as propositions that involve a condition and a consequent, were ignored by Aristotle and then developed by Stoic logicians. When the heritage of Ancient logic had been translated and transmitted into the Arabo-Islamic world, Arabic logicians attached some importance to the conditional propositions and the syllogisms comprising these propositions. Among those logicians, the most prominent and influential figure was Avicenna (Ibn Sina, d. 1037). As a philosopher and logician who presented himself as a disciple of Aristotle, Avicenna composed some great commentaries on Aristotle's works collected under the title '*Organon*'. Though his early commantaries are not extant today, we have his great philosophical opus, *The Cure*. In one of the logical sections of this voluminous book, *Kitab al-Qiyas* (*The Book of Syllogism*), Avicenna deals with conditional propositions and their relations so comprehensively that we hardly see in other's books before and after him.

In this paper, I shall examine Avicenna's remarks and considerations on conditional propositions and the oppositonal relations between them. I shall try to detect his Ancient sources and his influence on the tradition of Arabic logic after him. My aim is to show that Avicenna's theory of conditionals has some original aspects different from that of Ancient logicians and that he improved this theory significantly.

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Paul J.E. Dekker

A Cube, a Prism and a Fork of Oppositions

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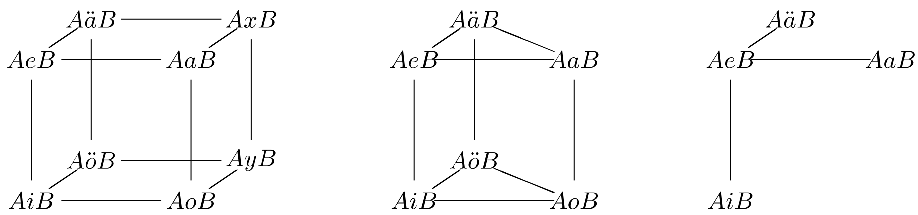
Auguste De Morgan (1846) has identified four categorial connectives (here ä, x, ö, y) which can be derived from the four familiar from Aristotle (e, a, i, o, respectively) by negating their subject terms. Hans Reichenbach (1952) has shown that the eight connectives form a cube of oppositions in a complete logical space. All four general propositions presuppose the three particular ones that they don't contradict, whence they are contraries; consequently the four particular propositions are subcontraries.

With the 8 logical connectives 96 syllogisms are valid, and they can be reduced, by conversions, to only 12. These, in their turn, can be reduced, by inversion and contraposition, to only one. Barbara can of course figure as the designated one, but anyone of the other schemes may do as well.

By means of a constraint (*extension*) from the theory of generalized quantifiers, the cube can be reduced to a *prism* of six oppositions which is, again, converse complete and, therefore, logically well-behaved. The skeleton of the prism is a fork of oppositions, e, i, a, ä, a set that is converse-complete again. Interestingly, these are the four that are generally lexicalized as appears from a recent cross-linguistic study (Keenan and Paperno, 2012). From a developmental perspective, one may speculate that i is the primary (non-)connective (the point), then comes e (the line), next a and ä (the fork), possibly o and ö (the prism), and finally, but not expectedly, x and y (the cube).



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The semiotic square and the knowledge organization: an application on popular songs classification

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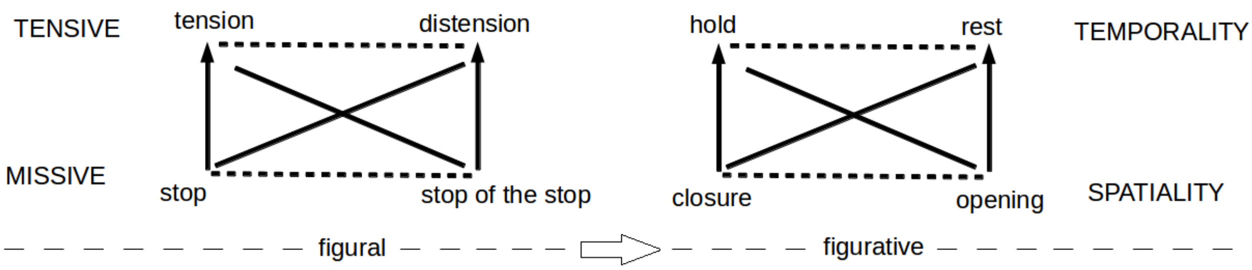
This work presents the results of using the semiotic square as a basis for creating a classification system for popular songs.

The semiotic square, as formulated by Greimas within the *School of Paris* Semiotics tradition, is «the logical articulation of a semantic category». Recent developments on *Tensive Semiotics* led by Claude Zilberberg and on *Songs Semiotics* led by Luiz Tatit, have successfully applied the semiotic square to the analysis of artistic objects, especially those in which is necessary to consider the *sensitivity* aspects as well as the gradations that exist inside a category, as occurs in the case of popular songs.

The analytical categories derived from *missive* and *tensive* levels allow to identify relations of tempo (acceleration / deceleration), rhythm (continuity / discontinuity) and syntax (melodic



profiles: passionate, thematic and figurative); beyond the narrative and discursive categories belonging to the lyrics. The figure bellow illustrates a schema of *tensive* and *missive* levels and the relations of spatiality and temporality:

The present research, conducted in the context of Knowledge Organization studies, identified the epistemological determinations for creating a classification system proper for popular songs. This model has been implemented through an ontology-driven prototype and applied to a set of 35 songs.

This talk highlights the importance of semiotic square as a tool for the Knowledge Organization field and its applicability for determining semantic categories suitable for classifying popular songs.

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Logical Geometries and Information in the Square and its Extensions

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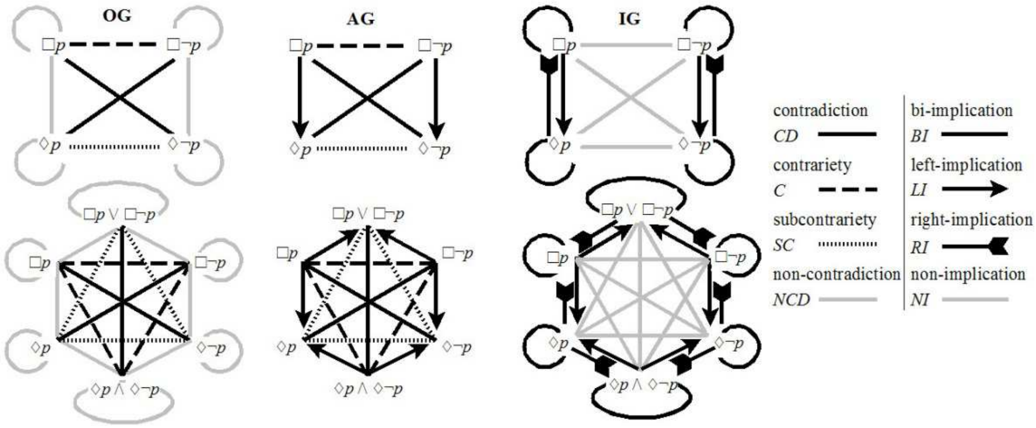
The main aim of this paper is to show that the well-known square of oppositions is more informative than almost all of its extensions. We first distinguish between a *geometry* (an abstract set of logical relations) and its corresponding *diagrams* (concrete visual representations of sets of formulas standing in those relations). The classical square is thus a diagram in the *Aristotelian geometry* (AG), i.e. the set of the contradiction (CD), contrariety (C), subcontrariety (SC) and subalternation (SA) relations. We show that AG exhibits a number of problems: its relations are neither mutually exclusive nor jointly exhaustive, and, most importantly, it is conceptually confused. We therefore introduce two new geometries:

the *opposition geometry* (OG) and the *implication geometry* (IG). The former inherits CD, C and SC from AG and replaces SA with the new relation of non-contradiction (NCD); the latter renames SA as left-implication (LI), and adds bi-, right- and non-implication (BI, RI, NI, respectively). We show that OG and IG jointly solve the problems of AG; furthermore, they have interesting historical precursors and exhibit a rich group-theoretical structure.
Next, we introduce a formal perspective on the informativity of the relations in OG and IG, based on the well-known idea of *information as range*: CD is the most informative relation in OG, NCD the least informative, and C and SC are in between; similarly, BI is most informative in IG, NI least informative, and LI and RI are in between. These results are highly intuitive, and cohere with the group-theoretical structure mentioned above.
Finally, we integrate all the above notions. In a first step, we extend the informativity account to the Aristotelian *geometry* by introducing the notion of winner. Given relations R $\in$ OG and S $\in$ IG, the winner of {R,S} is the relation that is most informative. It turns out that AG is hybrid between OG and IG (AG $\subseteq$ OG $\cup$ IG) in an informationally optimal way: we prove that a relation is Aristotelian iff it is a winner (modulo the cases of BI and RI, which can independently be accounted for). In a second step, we consider the Aristotelian *diagrams*, by examining the distribution of unconnectedness across these diagrams. Unconnectedness (U) is defined as the combination of NCD and NI, i.e. the two least informative relations of OG and IG, respectively. Alternatively, two contingent, non-equivalent formulas are U iff they do not stand in any Aristotelian relation. The Aristotelian diagrams avoid this minimally informative combination as much as possible. For example, neither the square nor the Jacoby-Sesmat-Blanché hexagon in the AG column below contain U, since their counterparts in the OG and IG columns do not contain NCD and NI in corresponding places.



Lorenz Demey & Hans Smessaert

Algebraic and Cognitive Aspects of Presenting Aristotelian Diagrams

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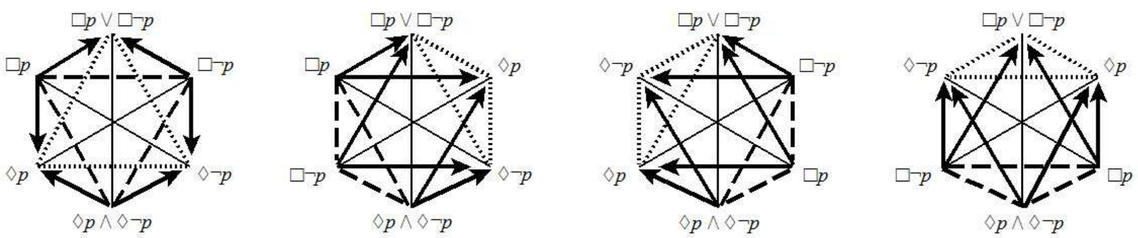
There has recently been much research on the Aristotelian square of oppositions and its various extensions. However, a question that has received little or no attention so far, is the following: for a given set of formulas, what is the 'best' way to visualize them and the

Aristotelian relations between them? For example, for a set of 8 formulas, one can construct a 2D octagon, but also a 3D cube. Is any of these diagrams 'better' than the other? In this paper, we will describe the beginnings of a systematic way of answering these questions.
We focus on Aristotelian diagrams whose set of formulas is closed under negation; hence a set of *2n* formulas can be seen as consisting of *n* pairs of contradictory formulas (PCDs) of the form (φ,¬φ). We also require that the diagrams have a central symmetry point around which all PCDs are centered. A simple combinatorial argument shows that the number of configurations of *n* PCDs is $n!\ 2^n$: there are *n!* permutations of these *n* PCDs, and each PCD can be put into the configuration in *2* ways, viz. as (φ,¬φ) or (¬φ,φ).
Consider the case of *2* PCDs (*4* formulas). There are $2!\ 2^2 = 8$ configurations of 2 PCDs. If we visualize these using squares, the 8 resulting squares are all symmetric/rotational variants of each other. The symmetry group of a square is the dihedral group D_4, which also has 8 elements. In sum, there is exactly one way (up to symmetry/rotation) of visualizing 4 formulas using a square. This shows that the square is the best diagram for this purpose.
Matters are not so simple in the case of 3 PCDs (6 formulas). There are $3!\ 2^3 = 48$ configurations of *3* PCDs. In contrast, the symmetry group of a regular hexagon is D_6, which has only 12 elements. Therefore, there are $\frac{48}{12} = 4$ ways (up to symmetry/rotation) of visualizing 6 formulas using a hexagon. For some sets of formulas, the differences between these 4 ways are highly relevant from a visual-cognitive perspective. For example, the figure below shows the 4 fundamental ways of presenting a Jacoby-Sesmat-Blanché hexagon for the S5-formulas □p, ◊p, □¬p, ◊¬p, □p∨□¬p, ◊p∧◊¬p; we argue that for most purposes, B. Tversky's cognitive principles for designing effective diagrams imply that the leftmost presentation is more effective than the other three. In contrast, for other sets of formulas, the differences between the 4 fundamental hexagons seem vacuous, which suggests that other diagrams might be more suitable. For example, the octahedron's symmetry group has 48 elements, and thus there is $\frac{48}{48} = 1$ way of visualizing 6 formulas using a octahedron.
If time permits, we will discuss the case of 4 PCDs (8 formulas), and compare the merits of octagons (2D), cubes (3D), and 16-cells (4D) for visualizing them. These considerations can be generalized to larger sets of formulas, using the notion of cross-polytope.



Jean-Pierre Desclés, Anca Christine Pascu

The "Cube of Oppositions" in the Logic of Determination of Objects (LDO) and in the Logic of Typical and Atypical Instances (LTA)

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The Logic of Determination of Objects (LDO) was presented in (Descles, Pascu, 2011). The primitives of this logic are the concepts and the objects. The concepts are operators in the sense of Frege (Frege, eds. 1971) and the objects are operands. The whole language of the LDO is an applicative system (Curry, 1958). The differences between LDO and the classical logic are: 1° objects in LDO are of two kinds: fully (totally, completely) determinate objects and more or less determinate objects; 2° objects in LDO are typical and atypical; 3° the duality between extension and intension of a concept is not kept. Based on the theory of typicality developed inside the LDO, an extended system of quantifiers, the *star quantifiers*, is constructed. In (Desclés, Pascu, 2012) the relation of this system with the Aristotle's "square of oppositions" is presented.

To account for the distinction between typical and atypical instances of a concept, (they all belong to the *expansion* or to the *extension* of this concept), it must be introduced the *intension* of this concept and articulate it to its *expansion* and its *extension* in such a way that one can describe atypical objects among the more or less determinate objects falling under this concept. The whole problem of typicality/atypicality led us no longer considered the duality between extension and intension (according to the law known as Port Royal law) (Descles, Pascu, 2011).

Ontologies of domains are structured networks of concepts and of classes of objects. Generally, in these ontologies, the problem of typical/atypical is not considered. Inside these ontologies only some objects are treated as exceptions without doing a deep "logical" analysis (especially the analysis of intensions) establishing that an object must be considered as an atypical object inner to the category or as an object on the edges of the category, and so "almost belonging" to it but not "belonging entirely".

For this reason, it is justified to make more complex the whole problem of categorization by taking into account objects which being no longer atypical, are nevertheless on the external outer edges of the category, so apprehended as being related to the category but no longer belonging to it. That is why, the Logic of Typical and Atypical Instances (LTA) (Desclés, Pascu, Jouis, 2013) was proposed.

The LTA is a version of LDO given a finer categorization among instances of a concept: *typical*, *atypical* and *borderline*.

In this paper, according to the idea of "borderline", we extend the system of quantifiers of LDO by introducing *borderline quantifiers*. These quantifiers are the LTA's quantifiers. They are added to the "cube of star quantifiers" and studied in relation to the negation and to the hexahedron obtained by extension of Blanché hexagon.

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Square of Opposition and Existential Assumptions in late Scholastic tradition

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In the ancient and medieval logical tradition, no logical theory was possible without being embedded into a semantic framework with strong ontological roots. In particular, in medieval logic the subject term in a proposition has a referring function, in fact it stands for (*supponit pro*) the place of each of the individuals of whom it is predicated, so the term is also known as distributed or taken in all its extension. A proposition is said to have existential import if the truth of the proposition requires a belief in the existence of members of the subject class. The particular affirmative **I** and particular negative **O** propositions (sub- contraries) have existential import; they assert that the classes designated by their subject terms are not empty[3]. There are at least two reasons why existential import is accepted, and are explained by the assumption that there are two types of theories of truth with respect to predication in the Middle Ages. The facility of both these medieval theories of truth is of type correspondentist type:

1. An affirmative categorical proposition is true only if an individual property signified by the term predicate inheres in the thing actually referred to by the term subject. So i.e., the sentence 'Aristotle is white' is true only if something like the whiteness (the property of being white) actually inheres in Aristotle;
2. An affirmative categorical proposition is true only if its terms which act respectively as subject and predicate refer to the same thing. So e.g. the sentence 'Aristotle is white' is true only if a man named Aristotle, the referent of the subject 'Aristotle' is a white man.

This way of understanding the truth of a proposition is exposed to different kinds of objection, in particular if the subject of categorical proposition is empty. Generally, medieval

[3] The assumption that **I** and **O** have existential import is rarely made explicit; rather it is taken for granted by most logicians, ancient, medieval, and modern alike.

logicians in this case, usually assume that the subject simply refers to nothing (*pro nullo supponit*). But this fact has some counterintuitive implications.
The aim of the talk is to analyze the answers given to this question in the early sixteenth century by some authors who were still discussing topics of scholastic logic: Johannes Eckius, Robert Caubraith, Augustinus Niphus. The crucial point of their investigations concerned a revision of relations of opposition, equipollence and especially conversion (in this view some authors had rejected the rules given by Peter of Spain).

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Antonino Drago

From Aristotle's Square of Opposition to "Tri-Unity's Concordance": Cusanus' Non-Classical Arguing about God

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According to Cassirer, Nicolas of Cues (1401-1464) was the first philosopher of modern knowledge since he introduced the human mind to the infinity, and moreover he searched for a new logic. In fact, he contrasted the "Aristotle's sect", since their two laws of the excluded middle and non-contradiction manage one faculty only of our mind, the *ratio*. In order to conjecture about infinity and wholeness he suggested applying rather the *intellectus* through a "coincidence of the oppositions", which, if intended in classical logic, squeezes the square of opposition to a segment of the two quantifiers. But according to Cusanus this coincidence constitutes a process of *transcessus*, named a *coniectura*, whose result overcomes the previous theses. He claimed to obtain by such a method more appropriate names of God. His book of 1460 is titled by a new name, *Possest*, which represents the coincidence of the two opposite aspects of the reality, the possibility (*Posse*) and the actuality (*est*). The book of 1462 is titled by a further new name, *Non-Aliud* (Not-Other). In the chapter 19 he contests Aristotle's classical logic - i.e. the *ratio* arguing through the square of opposition on the Being -, since it is unable to overcome the opposition of other and not-other. In his last two years, by re-elaborating *Possest* he obtained the new names *Posse* and *Posse ipsum* (1464).
By including the word *posse* the first and the last names both belong to modal logic. It is difficult to formalize them since the formalization of the (predicate) modal logic is disputable, even more when the identity (*ipsum*) is added. However, owing to the translation of modal logic *via* S_4 model, all the above names can be considered in intuitionist logic. The second name is a double negation that Cusanus stated to be not equivalent to the corresponding affirmative word, *Idem.* This fact means the failure of the double negation law; hence this name belongs to the intuitionist logic. It is easy to recognise in Cusanus' writings a great number of doubly negated propositions of this kind, included the definition of a *coniectura*, hence, they too belong to intuitionist logic. Moreover, he claimed to be

arguing in a rational way. Indeed, some of the doubly negated propositions compose *ad absurdum* arguments. By summarizing through 20 “Propositions” the discursive book *Non Aliud* he sketched a concise theory; which results to be organized in an alternative way to the deductive one. Furthermore, he repeatedly enounced the principle of sufficient reason, whose application converts the entire intuitionist square of opposition in the classical square. In sum, he introduced in an extensive way a new arguing in non-classical logic, although he did not formalize it. However, his highest motivation was not to obtain an intuitionist square of opposition, rather to characterize the “concordance of the Tri-unity” (an oxymore, according to classical logic). He obtained the “most accurate” name: “The not-other is no other than the not-other”; this proposition has no rival in expressing at the same time the three Persons and their Unity. In other terms, he was interested not in a four-fold logical structure, as Aristotle’s square is, rather in a super-threefold logical structure (i.e., three elements together with their unity). A possible transformation of Aristotle’s square of opposition into the latter structure is suggested as a specific conversion from classical to intuitionist logic. It may formalize at best Cusanus’ method of conjecturing through the coincidence of oppositions.

Laurent Dubois

Opposition² : a Logico-Divergent Approach of the Classical Aristotelian Square

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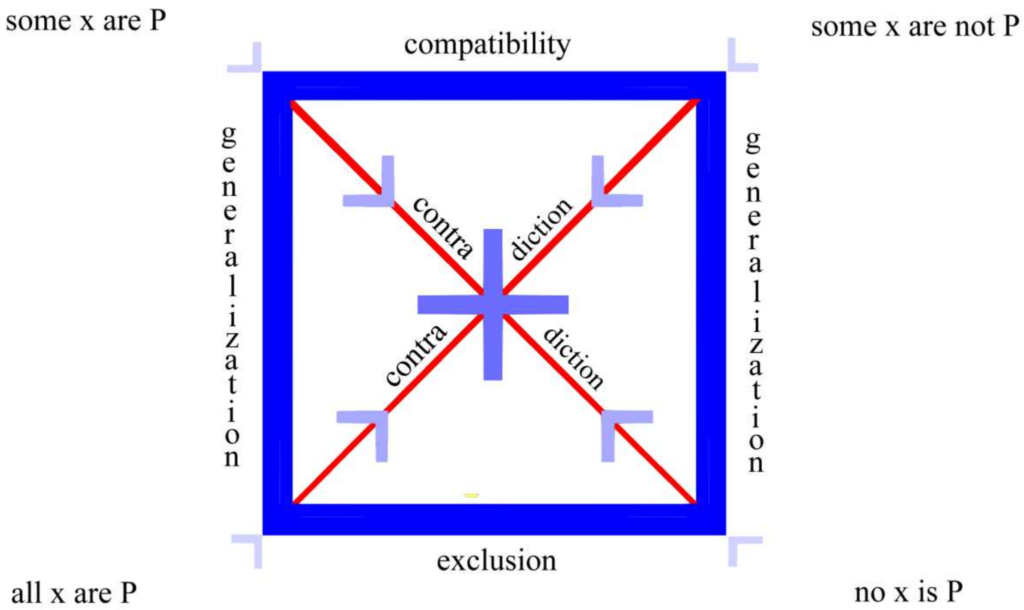
In this talk we will start from the following thought experiment: can the universe be aware/conscious of itself and how could it be so? Would this congress not be the best opportunity for the Universe to do its existential “coming out”?
What are the semantical and metaphysical consequences of a positive answer to the initial question? Does it also have any consequence on Logics in general and on the reading and the interpretation of the Aristotelian Square of Opposition in particular? Or is it the converse?



Precisely, as there are two ways to answer the initial question, we will see that there are two opposite readings of the Aristotelian Square: a deductive one (classical) and an inductive one (here illustrated).
Finally, we will define and explain the notion of Logico-Divergence.

Marie Duží

Squaring the Square of Opposition with empty concepts

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Though the traditional doctrine of the Square of Opposition remains a staple of logic to this day, it has been severely criticized in recent decades. The problems of the Square stem from using *empty terms in subject position*. Consequently, it has been argued that the relations embodied in the Square mostly disappear, because they are not logically valid without the existential commitment pertaining to concepts in subject position being satisfied. In this paper I am not going to trace the development of the doctrine from Aristotle to modern logic, nor am I interested in historical scrutiny, trying to coherently explicate what Aristotle or whoever else meant by the Square. Instead, I wish to introduce two consistent interpretations of the Square that include empty terms in subject position.
First, I improve on Strawson's attempt to rehabilitate the Square in terms of a logic of partial functions that complies with the modern notion of logical entailment and at the same time preserves the traditional doctrine. This explication consists in applying the *topic-focus* articulation criterion, according to which the topic of a sentence triggers a *presupposition* while the focus triggers merely an *entailment*. Thus, for instance, there are two readings of the sentence "Every *S* is *P*". Either the topic of the sentence is the subject term '*S*'. Then the sentence *presupposes* that there be some *S*s. In other words, if there are no *S*s then the proposition is *not true*; rather, it does not have any truth-value. Or, the sentence is about *P*s, claiming that the set of *P*s includes all *S*s. On this reading the sentence *merely entails* that there be some *P*s. Since the issues of topic/focus articulation and presupposition/entailment

have been recently dealt with in Duží (2012) and (to appear), the main goal of this paper is to systematically consider the second possible interpretation, which is one in terms of the modern *theory of concepts* as developed by Pavel Materna within the framework provided by Transparent Intensional Logic (see Duží et al (2010, §2.2), and also Materna (2004)). I am going to show that when we explicate Aristotle's subject-predicate propositions in terms of *concepts' intensions* rather than their extensions, then the Square remains valid even when empty concepts are included. On this interpretation the sentence "Every *S* is *P*" is to be read as "The concept *S* is subsumed by the concept *P*", or "the intension of *S* contains the intension of *P*", while the sentence "Some *S* is *P*" is to be read as "The concepts *S* and *P* are compatible". In addition I show that this interpretation also lends itself to a *modal* explication. Thus, for instance, the (A) proposition "Every *S* is *P*" can be read as follows. "*Necessarily*, *if* there are some instances of *S* then all these instances are also instances of *P*". The subaltern (I) takes the form "*Possibly* there may be instances of *S* that are also instances of *P*", which is true even in case of both *S* and *P* being empty concepts.

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Patrik Eklund

A Lative Logic view of the Filioque Addition

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The Filioque Addition in the Creed appears centuries before the Arian controversy, which culminated in the Toledo council 589. The role of the Son in proceeding of the Holy Spirit from the Father came to be a debate instrumented by Augustinus in his fight against Arianism and their ``procedit a solo Filio'' view of the Trinity (Niculcea, 2010). From logic language point of view, it is not immediately defendable to read ``Patre Filioque'' as ``Father and Son'', where 'and' is conjunctive, but indeed reading ``qui ex Patre Filioque procedit'' as ``comes from the Father and proceeds through the Son''. This removes the 'and' as a conjunction between `Father' and `Son', since the reading is then more equivalent to ``coming from the Father, [and] proceeding through the Son'', and from that the Trinity would be explained as if the Spirit isn't `given' until through the Son (St. John 7:39). The Greek ``ekporeuomenon'' should be seen in relation with the Aramaic ``npq'' and the Latin ``procedit'', e.g. as in St. John 15:26, and they are slightly different in respective languages.
Other attempts, trying to explain existence and Trinity, use predicates and existence symbols in the Aristotelian tradition, leading to ``mathematically heretic'' formulations e.g. as in Sabellius' arguments or efforts to encode the Shield of the Trinity in first-order logic.

Whereas Augustinus didn't debate Aristotelian logic, this debate opens up centuries later when Thomas Aquinas rightfully points out that Aristotle didn't try to explain the difference between being and existence. Clearly, Aristotle was not affected by the Logos as Thomas Aquinas was, but Aristotle saw the potential danger in using self-referentiality. We could say that Aristotle was very ``illative'' and clearly he was very ``unsorted''. Univocality and equivocality concerning *names* becomes important in *Summa Theologica*, and these names implicitly respect sorts. The Thomism view on the difference of being and existence is discussed also in (Basti, 2001), where *essence* is symbolically introduced, and is required to be an ingredient in existence. Symbolism can be strictly formalized in various ways, and we mainly follow the category theoretic and lative logic approach in (Eklund et al, 2014).

In Christianity, the role and notion of Church may also invite to say that Logos, as written in language, proceeds through the Church, includes Sacraments, and embraces teaching of Faith. This proceeding of Spirit and Logos, respectively, through Son and Church, is formulated in language, passes over time through translations and controversies, and ends up in different formulations. Is it perhaps so that we may not need not be so much ecumenic about the ``Spirit proceeding through'', since there is actually no real dispute in this matter, as compared to what we need to be about the ``Logos proceeding through''?

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Luis Estrada-González

An application of quaternality theory to topos logic

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In Walter Helbig Gottschalk's theory of quaternality, every involution in a mathematical system *M* (negation in classical zero- and first-order logic, complementation in the theory of classes, complementation and conversion in the calculus of relations, etc.) gives rise not only to a theory of duality, but also to a theory of quaternality. The main idea is that involutions serve to obtain not only a dual system of *M*, but also opposite and co-opposite systems of *M*, which can be arranged squarewise. Many well-known formal structures, like the quantificational and the modal decorations of the square of opposition, are particular cases of the square of quaternality.

Gottschalk presented quaternality bottom-up, from the special case of zeroth-order classical logic to the more general case of algebras. This has an evident benefit: It makes intuitive the idea of quaternality from well-known cases. In this paper I present Gottschalk's theory of quaternality but from a top-down perspective based on category theory and with applications to topos theory, especially to topos logic.

First, I provide the relevant definitions of the theory of quaternality in a general, categorial setting. Then I apply this to Ω, the object of truth values in a topos. The standard understanding of Ω, $_S\Omega$, is that it is a Heyting algebra. However, $_S\Omega$ can be factorized into the categorial structure, which is essentially equational, and the labeling of that structure, which is essentially a certain Skolemization of that equational structure. The crucial part then is defining a relevant involution, which in this case is a re-labeling morphism, which gives a different Skolemization for (the equational structure of) Ω. I show how quaternality allows thus at least other three readings of the categorial structure of Ω, with special attention to the case of dualization. When the operations of Ω are dualized, the logic induced is not intuitionistic but paraconsistent and one obtains thus Mortensen and Lavers' notion of complement-topos. Hence, the categorial structure of toposes also supports paraconsistency (the dual connectives of a topos are presented separately in an appendix). After that, I discuss with more detail the mathematical and philosophical import of quaternality for the internal logic of a topos, with special emphasis in the question whether the resulting kinds of toposes can be arranged as a suitable decoration of a square of opposition.

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S. Benjamin Fink

Oppositions in Introspective Disputes & Phenomenology

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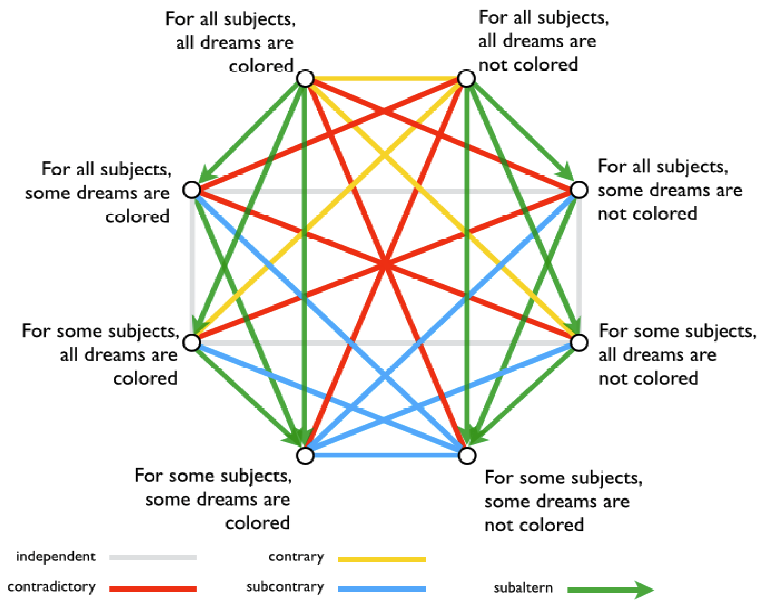
Are dreams colored (Schwitzgebel et al. 2006)? Do propositional attitudes have a unique phenomenal character (Bayne and Montague, 2011)? Do phenomenal properties represent (Tye 1992, Levine 1995, Block 2003)? There are considerable disputes about such matters of experience, which undermine the reliability of introspection in the eyes of some (Dennett 1991, Kriegel 2007, Bayne and Spener 2010, Schwitzgebel 2011): If introspection were a reliable method, these disputes should not arise; but they do. Ergo, introspection is not reliable. But disputes require contrary or contradictory statements. Do introspective statements stand in such relations?

An analysis of experiential statements leads to an octogonal shape of opposition, which does not smoothly map onto the shapes discussed by Hacker (1975), Béziau (2003), or Moretti (2004). It does, however, resemble a shapes discussed by Buridan in *Summulae de Dialectica* (cf. Read 2012).



If we look at those statements that are justifiable by introspection alone (i.e. are about experiences which are introspectively accessible to oneself), we see that these only stand in subcontrary opposition. I argue that there are therefore no genuine introspective dispute, and that what appears to be introspective “disputes” could just as well be indicators for heterogeneous experiences: Some dream in color, and some don't.

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The Orthomodular Square of Opposition and the Many Worlds Interpretation of Quantum Mechanics

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As it is well known, quantum mechanics faces serious difficulties in order to interpret the meaning of possibility which arises from the or- thodox formalism of the theory. In [1, 2, 7] we developed a scheme which allowed us to discuss within the same structure both actual and possible properties. Following this line of research and taking into ac- count the logical structure of quantum theory, we continued our analy- sis considering the Aristotelian Square of Opposition in Orthomodular Structures enriched with a monadic quantifier [3]. In [6] we provided an interpretation of the Orthomodular Square of Opposition expos- ing the fact that classical possibility and quantum possibility behave formally in radically different manners. In this work we attempt to analyze possibility in the Square of Opposition taking into account the so called many worlds interpretation of quantum mechanics [4, 5].

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Tzu-Keng Fu

On Institutional Contradiction and Organizational Plurality

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Nowadays many scientific researches are done within collaborative groups. Many epistemic activities have to be analyzed in the group or community. Apart from collective intentionality, a distinction between different categories of human agents as individuals and groups, which focuses around particular epistemic interests, has to be studied properly. John Searle in his work, *The Construction of Social Reality*, has given a general theory of social institutions. This theory is constructed in the course of particular investigations, having the heuristic purpose of facilitating consideration for the ontology of economics. Searle's concern is that from brute facts humans have the ability (with collective intentionality) to create institutional reality, which is a special case of social reality, such as money, government, marriage, and so on. This ability of human beings is an extension of more basic biological phenomena such as the ability of engaging in cooperative behavior and their innate capacity for linguistic symbolism.

Our viewpoint is that Searle's theoretical framework of institutional reality which provides us an analytical perspective on the organizational knowledge creation deserves certain interdisciplinary considerations on logic and cognition, in particular to the cross-cultural studies. First of all, we elaborate how "conceptual-metaphoric intelligence", the term that we coin to refer the ability of creating institutional reality can contribute to Searle's general theory of social institutions – on status function and the general logical form of the imposition of status function. All theoretical issues are worked out by applying Joseph Goguen and Rod Burstall's institution theory that has been widely discussed in theoretical computer science and mathematical logic. Secondly, we envisage the coexistence of multiple, different knowledge creation within an organization, which manages the multiplicity and diversity of epistemic activities, from the perspective of organizational theory. Following this, a certain sense of pluralism in organizational knowledge creation could be formulated, in an interdisciplinary manner. Finally, we argue that the underlying logical script of epistemic activities within organizational knowledge creating processes has been introduced by Jáskowski's problem.

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Robert L. Gallagher

Being and contradiction in Aristotle's *Metaphysics* and *Physics*

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Contradiction (*antiphasis*) characterizes processes in which being as substance comes-to-be or perishes, according to Aristotle's theory of substantial change. "The change not from a subject to a subject is coming-to-be through an *antiphasis*" (*kat' antiphasin*), Aristotle says in *Phys.* E.1.[4] One type is "the coming-to-be in an unqualified way from the non-being (*to mē on*) to substance."[5] Based on *Phys.* E.1, coming-to-be involves contradiction, the opposition between the previous non-being of a subject to its being as substance. That is a topic in *Met.*Λ.2, which says: "Everything comes to be from being, from being in potentiality, and from non-being in actuality."[6] Contradiction also governs perishing, The *Physics* text continues: "the change from a subject to not a subject is perishing, if unqualified, the change is from substance to the non-being (*to mē eînai*)."[7] *Phys.* E.1 may cast some light on the study of being qua being, which concerns "that which is peculiar to being,"[8] for one thing that is peculiar to being is that it emerges from non-being, and perishes into non-being. The contradiction between the two ends of the processes of coming-to-be and perishing structures reality, and contributes to our understanding of the Law of Non-Contradiction (LNC). For if it were not true that "it is impossible for a being [subject] to both exist and not exist at the same time,"[9] then contradiction would not be able to play its proper role in coming-to-be and perishing, for if LNC were not true, and something could "exist and not exist at the same time," then the non-being of substance would be cotemporaneous with the being of substance, and one would wonder whether substance could come-to-be or perish, at least under Aristotle's model of those processes.

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Katarzyna Gan-Krzywoszyńska and Piotr Leśniewski

Rationality as Indifference. On a Love-Hate Hexagon in the Foundations of Humanities

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> Indifference, to me, is the epitome of evil. The opposite of love is not hate, it's indifference. The opposite of beauty is not ugliness, it's indifference. The opposite of faith is not heresy, it's indifference. And the opposite of life is not death, but indifference between life and death.
> Elie Wiesel

[4] 225a12-14.
[5] 225a15-16.
[6] *Met.* 1069b18-20.
[7] 225a17-18.
[8] *Met.* Γ.3.1004b15-16.
[9] *Met.* Γ.4.1006a3-5.

If it is presupposed that such questions as *Why did agent X perform action A?*, or *What was the goal of action A performed by agent X?* are acceptable and justified within the framework of the humanities, it seems reasonable to investigate the relevant assumption of rationality. These two questions lead to a corresponding (anthropological) model of the human being and his/her actions. In consequence, this is also a model of the various interpersonal and social relationships taken into account (and/or ignored) in a given anthropological model. In terms of these relationships, special attention deserves to be paid to love and hate. It is presupposed that love is a relation between two different persons. If (person) *X* loves (person) *Y*, then *X* desires *Y*'s good and takes effective steps to secure it. Hence, love is irrational since *X* is interested in maximizing someone else's good, but not in his/her own. One may say then that "love enslaves". Therefore, appropriate (logical) hexagons of love-hate oppositions are introduced here. Moreover, we include in our hexagons corresponding concepts of rationality as indifference and of responsibility. At the same, important issues are raised related to systematic research on so-called *non-rationalities* (i.e. irrationality and counterrationality). The aim of our paper is to investigate these by means of oppositions within a logical hexagon.

José David García-Cruz

From the Square to Octhaedra

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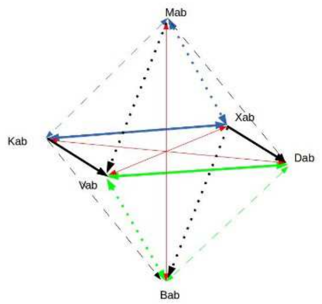
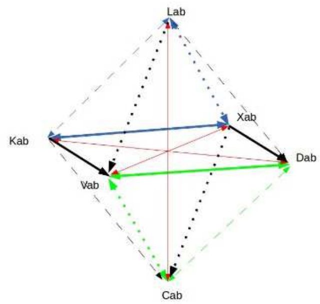
In (1972) Colwyn Williamson develops a comparison between propositional and syllogistic logic. He outlines an interpretation of the traditional square of opposition in terms of propositional logic, that is, the sentences corresponding to the nodes of the traditional square can be represented with propositional logic operators. His goal is to present a twofold square that preserves the truth conditions of the relationships between the formulas.

We present two octhaedra inspired in this twofold square. The octhaedra have an intersection in the contradictory formulas, and hold the basic relationships of the traditional square of opposition. These polyhedra keep also the traditional rules of immediate inference: conversion, equipollence, obversion, etc.

Our goal is threefold: first, to interpret this relations in a programming language, second, to develop a relationship between Williamson's squares, and third, to generate two new squares to complete the relations of the octhaedra. The programming language used is *Python*. We design a program that could accept as input formulas in polish notation, and generates as output a truth value, the truth value of the formula. The core of this program is the definition of the Boolean operators, we define this operators as functions that receive two variables and give a truth value. The relations of the octhaedra could be represented as formulas, and our program could validate these relations. The following diagram explains how we can mix the Williamson's squares to get these polyhedra.





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The Mereological Square

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By mereological square I mean a square of opposition which is edged by predications of the following form:

A-corner: All (proper) parts of x are F

I-corner: Some (proper) parts of x are F

E-corner: No (proper) part of x is F

O-corner: Some (proper) parts of x are not F

In this paper I shall show that the mereological square features different logical relations depending on x's standing for different kinds of wholes and I shall explore some such relations of particular interest.

Following Burkhardt (1989) and Burkhardt/Dufour (1991), I distinguish between the following three different kinds of wholes:

1) essential wholes, whose parts are not separable (example: universals);

2) aggregates, whose parts are all separable (example: heaps);

3) integral wholes with some parts which are not separable and some parts which are separable (example: human beings).

I distinguish between:

a) Mereological squares edged by (counter-) predications of a property to parts of a universal.

b) Mereological squares edged by (counter-) predications of a property to parts of a heap.

c) Mereological squares edged by (counter-) predications of a property to parts of an integral whole.

Given (a), the following is the case:

i) Contraries are not both false and subcontraries are not both true iff the property is the one which is defined by the universal.

Given (b), the following is the case:

ii) Contraries are always both false and subcontraries are always both true.

Given (c), the following is the case:

iii) The mereological square is edged by (counter-) predications of properties to the *inseparable* parts of an integral whole iff contraries are not both false and subcontraries are not both true. The mereological square is edged by (counter-) predications of a property to the *separable* parts of an integral whole iff contraries are both false and subcontraries are both true.

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Raffaela Giovagnoli

Aristotle, Frege and "Second Nature"

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Aristotle proposed a "naturalistic" epistemological perspective that rests on some fundamental notions:

- perceptual judgment (passivity and activity),
- simple propositions (subject and predicate)
- complex propositions (syllogisms)

As it is well known, the "Square of Opposition" provides the possibility of a fruitful classification of reality that is made of things, species and genus

Frege introduced a new form of notation that is exemplified in his *Begriffschrift* and changed the Aristostelian square. The distinction between "function", namely the fixed part of an expression and "argument", namely the variable part of it, plays the fundamental epistemological role to indicate when the argument is "determinate" or "indeterminate". This very distinction is relevant for specifying a new notation of "generality", which differs from the Aristotelian one and rests on a substitutional strategy.

Frege's logic developed sophisticated logical relations among concepts that rest on the fundamental notion of "unsaturedness". Here, we can represent the falling of an individual under a concept by F(x), where x is the subject (argument) and F( ) is the predicate (function), and where the empty place in the parentheses after F indicates non-saturation.

He introduced a conception of judgment that entails a fundamental relationship with a "second nature". It means that we must recognize "true" judgments that derive from our belonging to a scientific tradition. It is a matter of our acquiring adequate conceptions of things and of the world that manifests its face to us. Starting from this background, McDowell and Brandom present two original views of the "second nature" which are subject to some criticisms.

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Back to Hegel and the right square

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Given that not all forms are propositional and some of them presuppose the validity of one-sided inferences, it will be seen that the probabilistic representation of the proper Aristotelian forms:

(A) pr(P*x*/S*x*) = 1,
(I) pr(S*x* & P*x*) > 0,
(E) pr(S*x* & P*x*) = 0,
(O) pr(P*x*/S*x*) $\neq$ 1,

is unsound. To advance the comprehension of oppositions I will appropriate Hegel's (p607, p661, p671, p789) use of two-sided inferences; where the variables of the functions are in a relation of difference to their complements. This will mean the likelihood of the two-sided use of inferences to calculate probabilities in its relation to the one-sided use of inferences will have a logical basis. To this end the modal forms of necessity, possibility, impossibility and contingency for the relation of grounded and ungrounded probability functions will constitute the *right square*:

$$\Box\ \forall Pr_g \forall Pr_u \forall E \left(Pr_g(E) > Pr_u(E)\right)$$
$$\Diamond\ \exists Pr_g \exists Pr_u \forall E (Pr_g\,(E) \geq Pr_u(E))$$
$$\neg\Diamond\ \exists Pr_g \exists Pr_u \forall E \big(Pr_g(E) \geq Pr_u(E)\big)$$
$$\triangledown\ \exists Pr_g \exists Pr_u \exists E \big(Pr_g(E) = Pr_u(E)\big)$$

It will follow that a simple proof structure for *exclusive or* $\vee_e$ *or inclusive or* $\vee_i$ can be given. This will be coherent with the binomial for a subject's credence given the sample of the function, so corresponding to what can be predicated for the probabilities of events.

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Anne Hénault

The Square of Opposition in All its States

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Semiotics have used and misused the square of opposition, continuously adapting this figure to its descriptive and analytical problematics. Let us follow the main avatars of the square of opposition in its semiotic story, from the moment when, echoing the work of R. Blanché, it was thrust upon world semiotic research by A.J. Greimas (1970), up to the most recent publications of the Cercle Sémiotique de Paris.

After the 1970-1975 phase during which all semiotic work appeared bound to lead to the square of opposition, Alain de Libera, F. Nef and a few logicians formulate the first sound epistemological criticisms. In turn, the most convinced semioticians make known their own reticences towards this "Constitutional Model of Signification", (*Le carré sémiotique*, March 1981)

Du sens II (1983) still places great emphasis on the square of opposition. Conversely, the last book personally written by A.J. Greimas *De l'imperfection (1987)* steers radically clear of the square henceforth treated as an almost trivial "machinetta". Then Semiotics of the passions (and of the sensitive in general) promote, amidst other notions, the concept of "tensivity" which is posited as incompatible with the discontinuous Semiotics of action for which the square appears as the keynote figure: by then, the square of opposition tends to be replaced by some curvilinear graphs (1993-2010).

Well then, in 2011, with *Corps et sens* (Presses Universitaires de France) one of the champions of this supposedly continuist alternative to the square of opposition, J. Fontanille, ends up in a series of composite figures where, certain squares of oppositions surge up again amidst the curvilinear graphs.

This irrepressible constancy of the semiotic square calls for a renewal of the interrogations on the place of the square of opposition in the theories of signification. We will therefore question its apparently inescapable efficiency for Semiotics.

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Aristotle on Deduction and Empty Terms

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There is a controversy regarding empty terms in Aristotle's system of deductions. Some commentators deny their relevance, while others allow them. This talk will show that empty terms are not a problem for Aristotle, as they are made possible only in some restrictive cases. Let us focus on the first syllogistic figure in the *Prior Analytics* with four conclusions accounting for the four kinds of deductive argument:

BARBARA Every B is A (major) <u>Every C is B (minor)</u> Every C is A	CELARENT No B is A (major) <u>Every C is B (minor)</u> No C is A
DARII Every B is A (major) <u>Some C is B (minor)</u> Some C is A	FERIO No B is A (major) <u>Some C is B (minor)</u> Not every C is A

The conclusion of BARBARA is true, if C is a non-empty term; for instance, the truth of 'every magnitude is continuous' implies the truth of 'some magnitudes are continuous'.
The conclusion of CELARENT is true, if C is either an empty term or a non-empty term of which A is not predicated; for instance, the truth of either 'no goat-stag is an animal' or 'no animal is a plant'.
The conclusion of DARII is true, if C is a non-empty term; for instance, the truth of 'some numbers are even'.
The conclusion of FERIO is true, if C is either an empty term or a non-empty term of which A is not predicated; for instance, the truth of either 'not every centaur is an animal' or 'not every living being is an animal'.
Therefore, a denied conclusion, whether universal or particular, can be about an empty term, and this does not prevent the argument from being a deduction. Yet, Aristotle lets us know in the *Posterior Analytics* that we do not learn anything from such a deduction.

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Tableaus for Numerical Syllogistic

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Numerical syllogistic deals with numerical categorial propositions. We have twelve kinds of such propositions:

a) two kinds of general classical categorial propositions:

S**a**P	Every S is P.
S**e**P	No S is P.

b) ten kinds of numerical propositions, where *n* is a natural number - in the cases third and eight *n* is bigger than 0:

$S\mathbf{i}^{n}P$	Exactly *n* S is/are P.
$S\mathbf{i}^{\leq n}P$	At most *n* S is/are P.
$S\mathbf{i}^{<n}P$	Fewer S than n is/are P.
$S\mathbf{i}^{\geq n}P$	At least *n* S is/are P.
$S\mathbf{i}^{>n}P$	More S than *n* are P.
$S\mathbf{o}^{n}P$	Exactly *n* S is/are not P.
$S\mathbf{o}^{\leq n}P$	At most *n* S is/are not P.
$S\mathbf{o}^{<n}P$	Fewer S than n is/are not P.
$S\mathbf{o}^{\geq n}P$	At least *n* S is/are not P.
$S\mathbf{o}^{>n}P$	More S than *n* are not P.

In our presentation we describe natural and intuitive semantics for those propositions. We present also a tableau system of Numerical Syllogistic that allows to determine correct arguments. This system is sound and complete to the semantics. Thanks to that system we can estimate a minimal cardinality of a domain of model that allows to decide whether an argument is or is not correct.

We show also that Numerical Syllogistic is a generalization of Classical Syllogistic, since general classical propositions S**a**P and S**e**P can be reduced to propositions $S\mathbf{o}^{n}P$ and $S\mathbf{i}^{\mathbf{n}}P$, if n=0. Other reductions are still possible. Moreover other squares of oppositions than traditional are possible.

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Dany Jaspers

The Growth of Lexical Fields

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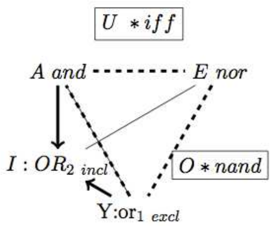
A systematic limitation on concept formation bars natural lexicalisation of both the U-corner and the O-corner of Blanché's (1953) logical hexagon (Jaspers 2012).



A plausible analysis for this state of affairs is that the foundational binary opposition (step 1) in the square and its offshoots is the E-I contradictory opposition. This fundamentum divisionis is arguably inviolable for further concept formation in the same lexical field. The lexical predicates in A and Y therefore form a secondary binary opposition, carved out entirely within I, which functions as their subuniverse (step 2). Combining either A or Y with E to form the disjunctive concepts of the U and O-corners is then nonnatural, as the resulting concept breaks out of the I-subuniverse.

The present paper will take it from there and show that this incremental perspective on the basis of two consecutive binary steps can be formalized. This will be done by proving that the basic set of natural operators of the propositional logic of natural language can be composed via gradual incremental elaboration on the basis of a single conceptual occupant of the E- corner, a revised version of Peirce's joint falsehood analysis (= NOR(P,Q) or Peirce's dagger). Though no further tool is necessary, it will be argued that the compositional content of *and* in natural language is more complex than the simplest composition in terms of Peirce's dagger. Specifically, it will be shown that it contains the meaning of *I : OR2 $_{incl}$* as a presupposition in addition to its asserted content *((P ↓ P) ↓ (Q ↓ Q))*. The same presupposition will be argued to be part of the meaning of exclusive *Y:or1* $_{excl}$. Although the above amounts to an asymmetrical growth pattern behind the four-cornered hexagon which starts from E, the orientation of this pattern is turned around when the system is viewed from the angle of lexicalisation and the growth of lexical fields in language acquisition. Thus, there is not only the well-known asymmetry which states that the lexical label of a negative is dependent, hence posterior to lexicalisation on the affirmative side (*n-or* is built on *or*), but also that the lexical item that the E-operator is the negative of, namely I, shares its lexical label with the item of the A-Y-pair that is lexicalised latest on the affirmative side. There is indeed evidence that in the case at hand *or* is acquired later than *and*. Interestingly, this inverse order of lexicalisation generalises to a host of other lexical fields.

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The Catholic factor in 20th century Square studies

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For most of the twentieth century, the Catholic Church put up stiff resistance to mathematics-based modern predicate logic (MPL), mainly because it was felt that MPL 'dehumanized' logic and because MPL took logic out of Thomistic theology and metaphysics, then considered to be the theoretical and ideological foundation of the Church. This battle was lost (Maritain, the Church's main protagonist, is not taken seriously by logicians), but, as has recently become known through the work of Horn and others in the context of the renewed interest in the Square, a few, relatively obscure, Catholics made serious attempts at 'saving' the Square, or at least at showing the hitherto hidden logical potential of the Square and its relevance for the study of human cognition. We will single out the American Paul Jacoby and the Frenchmen Augustin Sesmat and Robert Blanché, all three ardent Catholics. These three men have never been given proper recognition and have undeservedly slid into virtual oblivion. We present an account of their original contributions to the study of logic and cognition, along with their biographical details to the extent that we have been able to unearth them (which was no easy matter). We also sketch the general outlines of the worldwide Catholic cultural revival which took place between roughly 1850 and 1950 and which provides an essential historical backdrop to the work of these three men.

Priyedarshi Jetli

Knowing and Doing Wedded through Epistemodeontic Square, Hexagon, Cube and Hexagonal Prism of Opposition

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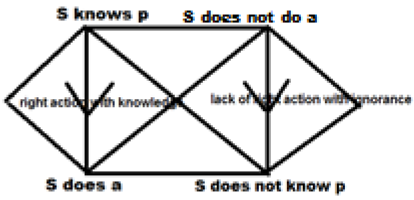
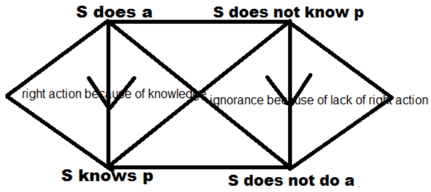
For Socrates, right action requires practical as well as theoretical knowledge. Deontic squares and hexagons of opposition have been constructed frequently. Epistemic squares and hexagons are not as frequent since the definition of 'knowledge' means that not knowing involves a disjunction of possibilities. However these squares and hexagons fail to display the link between knowing and doing. I display the link with two alternative squares and hexagons:



Epistemodeontic square and hexagon

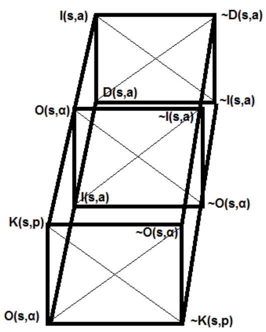


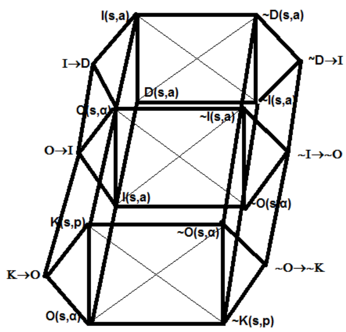
Deontiepistemic square and hexagon

I construct the epistemodeontic square of opposition beginning with the left subaltern in which 'S knows *p*' implies 'S does *a*'. The link between knowing and doing is established internally by making 'S is propelled into action by S's belief *p*' a necessary condition for knowing. The subcontrary relation shows that S may do *a* without knowing *p*. This is the possibility of right action without knowledge. Plato claimed in *Meno* that true opinion (which is not yet knowledge) was good enough for right action. The deontiepistemic square of opposition is guided by the intuition that 'S does *a*' only if 'S knows *p*'. Again I begin constructing the square with the left subaltern, 'S does *a*' implies 'S knows *p*'. The contrary relation makes it possible for S to not do *a* and yet know *p* (knowledge which would lead one to do *a*). This would account for *akrasia*, that is, one knows what the right thing to do is and perhaps even intends to do it and yet fails to actually do it.

In the epistemodeontic square the implication from knowing to doing itself is multi-layered and the square is a compact device to capture the main link between knowledge and ethical action. Propelling into action will imply that one ought to do action of type *α* given proposition *p*, this in turn will imply that one intends to do action *a* which is a token of type *α*, and finally this will imply that one actually does action *a*. For example, knowing that 12.4 percent of the world population is hungry may lead me to the knowledge of the proposition 'there is an unacceptable number of hungry people in the world', which, in turn leads me to the obligation of doing action of a type which will aid in removing hunger, this obligation in turn would lead to my intention of doing a particular action of starting a daily soup kitchen for 10 hungry people, and the intention would then lead to my actually running the soup kitchen daily. Two alternative expansions bring out all the chains of the causal links.

First expansion: Layered cube and hexagonal prism.





Second expansion: A cube is constructed with knowledge implying obligation square α at the top layer and the obligation implies intention square as the bottom layer. Then, the cube is expanded into a four dimensional tesseract adding intending implying doing. If 'ought implies can' is included then the tesseract is extended to a 5-cube. The hexagon would also be extended from a hexagonal prism to a four dimensional hexachoron to a five dimensional hexatope.

Spencer Johnston

Buridan's Octagon of Opposition & Ontological Commitment

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It is well known that in his writings on the modal syllogistic, John Buridan's theory of the syllogism both differs from and expands Aristotle's theory in a number of important ways. For example, in treating divided modal propositions of necessity and possibility Buridan expands the square of opposition to an octagon of opposition. Analogously to the square of opposition, a number of interesting and distinctly modal questions arise about the existential commitments of negative and affirmative propositions in the octagon of opposition.
These issues emerge because Buridan does not limit himself to only looking at singular terms that actually do or do not fall under a particular term. He also considers objects that can, or must, or only contingently fall under a particular term.
On Buridan's reading of modal propositions, `some B can be A' is true if something can be B (even if it currently isn't B or doesn't exist at all), that can also be A. Logically, this gives rise to a number of ontological questions in Buridan's octagon of opposition, that are analogous to the questions that revolve around the interpretation of the square of opposition.
The main question that this paper will address is: Does Buridan's modal logic commit him to the view that there are non-instantiated possible objects? More concretely, it seems that for `Some person can run' to be true, one needs to quantify over possible objects. Does this quantification then commit Buridan to the existence of possible people? We will argue that Buridan is not committed to any objects of this kind.

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Tomasz Jordan

Three opposition-forming negations as truth functional unary connectives

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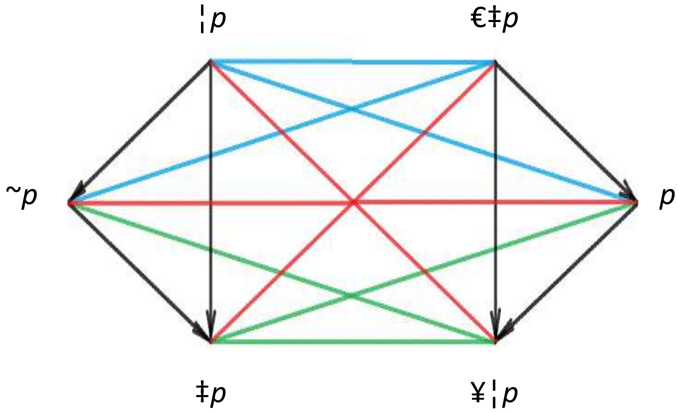
According to logical oppositions: contradictory, contrary and subcontrary, we can speak of three various sentential negations: standard or classical negation and two non-standard negations. I have obtained a classical-like propositional logic (CPL+2 for short) with such truth functional unary connectives: contradictory-forming negation (symbol ~), contrary-forming negation (symbol ¦) and subcontrary-forming negation (symbol ‡). It works by dint of an assignment based on four-valued truth tables that provide classical meaning of all standard logical connectives and validate all theorems of classical logic. Two extra truth values (separate: *true′* and *false′*) belong merely to the domain of this assignment whose range contains just two standard truth values.

The figure below comprises four symbols from the CPL+2 formal language: the above three connectives and a sentential variable *p*. Both the signs € and ¥ do not belong to CPL+2 but represent any of the negation operators and thus the following graph actually shows at once nine corresponding hexagons containing the sentence letter *p* and just the three connectives in question. Arrows mark the consequence relation, lines hint at the oppositions: red—contradictory, blue—contrary, green—subcontrary. Every relation presented by the mentioned nine hexagons of opposition (of the common form depicted here) is obviously expressed by a tautology in CPL+2.



A comparison of each of these non-standard negations to a certain paraconsistent or paracomplete negation should surely take into consideration that in CPL+2, firstly, all propositional letters range only over statements being either true or false, secondly and noteworthy too, every negation connective always provides a different truth value than its argument takes at the same time.

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Andreas Kapsner

On Guilt and Innocence

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Two concepts that are of obvious importance to criminal trials are the concepts of guilt and innocence. A clear analysis is of utmost importance, especially in common law countries where a lay jury will have to decide on the guilt of the defendant and should be adequately instructed about their task. It is the purpose of this talk to show that the square of oppositions can be employed to make important conceptual relations salient.

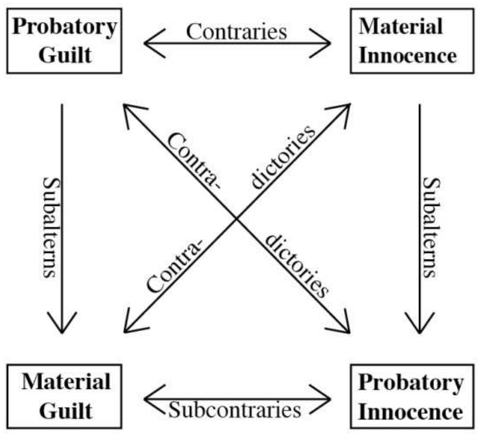
Many introductory texts to legal theory will advise the reader to avoid the common confusion between "not guilty" and "innocent". "Guilty" is the appropriate verdict if the evidence produced by the prosecution suffices to meet a given standard of proof. "Not guilty" is the appropriate verdict if the evidence fails to meet this standard. Innocence, on the other hand, is not an evidentially constrained concept. A person is innocent of a crime if she did not commit that crime.

The common law system is set up in a way that seeks to minimize the false conviction of innocent people. This explains the fact that in many trials the correct verdict will be "not guilty", even though the defendant is not innocent. It is indeed not the task of the jury to decide about the innocence of the defendant, but only to gauge the strength of the presented evidence.

Larry Laudan has recently suggested to stop trying to differentiate between the two conceptions by regimenting the uses of "guilt" and "innocence", and use the terms as the antonyms they are naturally taken to be. Instead, the two readings should be disambiguated by the tags "material" and "probatory". The decision that the jury has to make will now be construed as one between probatory guilt and probatory innocence, abbreviated as guilt_p and innocence_p (vs. the material variants guilt_m and innocence_m).

Laudan notes that "[t]here is a salient asymmetry between the two pairs of distinctions. It consists in the fact that a) while a finding of guilt_p sustains (fallibly) the assertion of guilt_m (that is, the legal system justifiably assumes that someone proved to be guilty is genuinely guilty), b) a finding of innocence_p (a "not-guilty" verdict) warrants no inference about innocence_m." ([1], p. 96)

However, this asymmetry turns into a quite pleasing symmetry once we arrange the items adequately. Indeed, the four concepts can be neatly mapped to the four corners of a square of oppositions:



As Laudan has noted, there is no implication from probatory innocence to material innocence, but there is one in the other direction. This is a somewhat idealized view of the matter, but it is the same amount of idealization that leads him to proclaim an inference from probatory guilt to material guilt.

The other logical relations that the square of oppositions indicates also hold between the four concepts. A logically trained philosopher might see this almost immediately, a lay juror may be well advised to think carefully through each of the depicted relations.

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Whether Aristotle applied octagonal calculator to compute his ethics?

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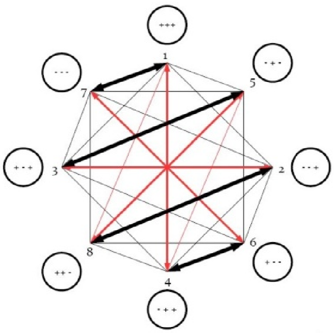

ACTIO	OBLIGO	NON-OBLIGO
A	+ + +	- + +
I	+ + -	- + -
O	+ - +	- - +
E	+ - -	- - -

The fact that ancients and medieval scholars used diagrams as a tool for demonstration in logical and didactic purposes is well-known. The question about the deepness of geometrical analogy remains still open. Whether the geometrical analogies were merely superficial or played an essential role in developing, deducing and formulating the conceptual foundation of metaphysics, natural philosophy and ethics? These patterns help us to understand the way how ethical concepts and vocabulary were deduced and logically related in ancient doctrines. We know (beside the classical square) that Aristotle must have seen in front of his eyes the natural-philosophical octagon (four oppositions: fire-water etc), which later inspired young Gottfried Leibniz. The research is a kind of conceptual-logical archeology that tried to detect verisimilar patterns-analogies in which Aristotle thought. Conceptual reading of the main peripatetic treatises on ethics, namely *Nicomachean Ethics* and *Magna Moralia*, which partly overlap, revealed that squaring of concepts was central methods in deducing Aristotelian ethics. At the same time it is important to make distinction between the context of calculation and the context of demonstration. The latter had a hexagonal appearance. It seems that Aristotle had three different calculators for separate realms of human inquiry. Every realm is implicated by a specific value-pair (true-false, goo-bad, right-wrong). The Porphyrian tree and the square of opposition was designed for the natural philosophy and ontological questions. Aristotle seemed to use for near-wisdom questions an octagonal abacus (the axiological calculator) that has been a two-dimensional variant of a logical cube. A (quasi-) logical hexagon was a simplifying demonstration tool for political-social questions (the practical calculator).

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Appendix

Relatio (EN II.7)	(≥)	(≤)	=	>	<	≠
Hardship	*confidentia*	*metus*	***fortitudo***	*confidens*	*ignavus*	*impotens impotentia* ***(ἀκρασία)***
Money	*accipio*	*dono*	***liberalitas***	*prodigalitas*	*avaritia, illiberalitas*	*impotentia*
Big money	*accipio*	*dono*	***magnificenti a***	not translated	not translated	*impotentia*
Esteem	*honore*	*infamia*	***magnitudo animi***	*elatus, elatio animi*	*pusillus*	*impotentia*
Public life	*gloria*	*ingloria*	missing	*ambitiosus*	*inambitiosus*	*impotentia*
Harm	*ira*	-	***clementia***	*iracundus, iracundia*	*lentitudo*	*impotentia*
Telling	*veritas*	*(falsum)*	***verax***	*arrogantia, arrogans*	*dissimulator*	*impotentia*
Social life	*voluptas*	*dolor*	***comitas urbanitas***	*scurra*	*rusticus*	*impotentia*
In public	*voluptas*	*dolor*	***facilitas***	*assentatio, assentator*	*morosus*	*impotentia*
Feelings	*pudor*	*impudor*	***verecundus***	*pavidus*	*impudens*	*impotentia*
Unworthy bad or good	*voluptas*	*dolor*	***indignatio***	*invidia*	*malevolentia*	*impotentia*
Corporal pleasures	*voluptas*	*dolor*	***temperantia***	*intemperantia*	-	*impotentia*

William Lewis Klein

Blood > Money > Information. The Pattern and Direction of Human History

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This paper presents human history in terms of system design, ultimately illustrating the pattern and direction of human history in a single graphic. The system is four-dimensional, consisting of time plus three human dimensions that intersect in every person: being a group member, being a pair partner, and being an individual. Each dimension is polar. At one pole is a foundational institution that provokes a universal problem; at the other pole is a counterbalancing institution that people long ago devised as a “solution” to that problem. The end result is three pairs of intersecting institutions that form the universal pattern of human societies. This pattern provides the skeleton upon which cultural uniqueness grows. In graphic form, the pattern serves as a lens for illustrating how the cultural expressions of humanity’s universal institutions have evolved over time in response to technology. This evolution occurred as societies responded to three great historical challenges: Survival, Progress, and Personal Freedom. People adapted to these challenges because success in each of the three dimensions requires a different set of skills, and they learned that the skills

associated with one particular dimension were more useful for meeting the historical challenge they were facing. In short, the relative influence of each of the three dimensions has not been equal over the course of history. Thus, the human story is the move from the struggle for Survival during the Reign of Groups (Agricultural Age) to the struggle for Progress during the Reign of Pairs (Industrial Age) to the struggle to expand Personal Freedom during the Reign of Individuals (Information Age). The human story is the move from Blood to Money to Information in a three-stage process that can be loosely viewed like phase states in physics. The resulting ontology is scalable and fractal; it unfolds at the global, national, local and personal levels.

Yaroslav Kokhan

Pragmatic Square of Opposition

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There is the hierarchy of norms that is quite analogous to the hierarchy of quantified prenex formulas. The simplest prenex formulas are the ∀–formulas and ∃–formulas; the simplest norms are the *imperative* norms ("*x* must do *A*") and the *permissive* norms ("*x* may do *A*"). Words "must" and "may" denote *pragmatic* acts, i. e., acts that any person can make with some sense.

The first pragmatic act — *assertion* — was discovered by Frege. We introduce two another pragmatic acts: *impeling* and *acceptance*. Let's take any sentence; if it's sense can be asserted, we'll call this sense a *proposition*; if one can impel someone by this sense to some actions, we'll call that sense a *request*; finally, if one can only accept or not this sense (as a fact or a convention), we'll call it an *admission*. Let's denote the sense of expression *E* by '<> *E*', assertion of proposition *A* by '<>– *A*', impeling by request *B* by '<>! *B*', and acceptance of admission *C* by '<>● *C*'. Every pragmatic act can be realized in many ways (with many kinds); for instance, an impeling can be an order, a question, an ask, a suggestion, a provocation etc. Hence every pragmatic act can be done only as a part of some pragmatic predicate having one of the next three forms: assertion predicate "*x* asserts for *y* by the method *F* that *A* holds":

F (*x*, *y* <>– *A*),

impeling predicate "*x* impels *y* by the method *F* to do *A*":

F (*x*, *y* <>! *A* (*y*)),

and acceptance predicate "*y* accepts *A* with respect to *x* by the method *F*":

F (*x*, *y* <>● *A*(*y*)).

We call *x* the *subject* or the *agent* but *y* the *recipient* of these predicates. Agent and/or recipient can be absent in some pragmatic predicates; we sign their absence by the empty letter '★'. We call every pragmatic predicate without subject *impersonal*.

Every assertion act can be hypostatized; as a result we obtain truth values *truth* and *falsehood*. Some accepting acts can also be hypostatized; as a result we obtain valuations “good”, “bad” and some other. But all the impeling acts and some of accepting acts are non-hypostatizable. They are dual, so there is the pragmatic square of opposition:

F (*x*, *y* <>! *A* (*y*))	—	*F* (*x*, *y* <>! ¬*A* (*y*))
|	×	|
G (*x*, *y* <>● *A* (*y*))	—	*G* (*x*, *y* <>● ¬*A* (*y*))

Here predicates *F* and *G* are not identical. Norms are exactly non-hypostatizable impersonal pragmatic predicates of impeling (imperative norms) and accepting (permissive norms), so there exists the normative square of opposition:

F (★, *x* <>! *A* (*x*))	—	*F* (★, *x* <>! ¬*A* (*x*))
|	×	|
G (★, *x* <>● *A* (*x*))	—	*G* (★, *x* <>● ¬*A* (*x*))

Przemysław Krzywoszyński, Jerzy W. Ochmański

On some oppositions between political systems: Aristotle, Montesquieu and Modern Democracy.

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The aim of our paper is to present triangles, squares and hexagons concerning political regimes, both in classical and contemporary political philosophy. Firstly, we will analyze Aristotle's triangles for good and bad political regimes; this is monarchy, aristocracy and politeia, and respectively, tyranny, oligarchy and democracy. Secondly, we will reflect on Montesquieu’s classification of systems of government based on principles that can be presented in a square: democracy, republic, monarchy and anarchy. Moreover, we would like to propose an extension of this square to a hexagon, adding historical regimes from 18thcentury Poland and Great Britain. In terms of contemporary research, we discuss some triangles of oppositions that will enable us to consider some conceptual paradoxes within the modern theory of democracy from a new perspective. These figures are constructed according to such criteria as voting and procedures for participation. We also postulate by means of contradictories, contraries and subcontraries a new perspective/insight on post-democratic reality.

Oliver Kutz

Shapes of Opposition in Conceptual Blending

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In this talk, we will introduce the formalisation of conceptual blending as it was given by Joseph Goguen (see e.g. [1]), and discuss in particular the oppositional shapes that arise in

the heterogeneous set-up, namely when constructing blendoids from input theories given in different logical languages.
Conceptual blending aims at creatively generating (new) categories and ontological definitions; this is done on the basis of input theories whose domains are thematically distinct but whose specifications share structural or logical properties. As a result, conceptual blending can generate new concepts and it allows a more flexible technique for theory combination compared to existing methods.
Our approach to computational creativity in conceptual blending is inspired by methods rooted in cognitive science (e.g., analogical reasoning), ontology engineering, and algebraic specification. Specifically, we introduce the basic formal definitions for theory blending, and show how the distributed ontology language DOL (see [4] for basic definitions and examples and [2] for the theoretical background) can be used to declaratively specify blending diagrams and study oppositional shapes, extending earlier work [3].

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Genevieve Lachance

Platonic Contrariety (enantia): Ancestor of the Aristotelian Notion of Contradiction (antiphasis)?

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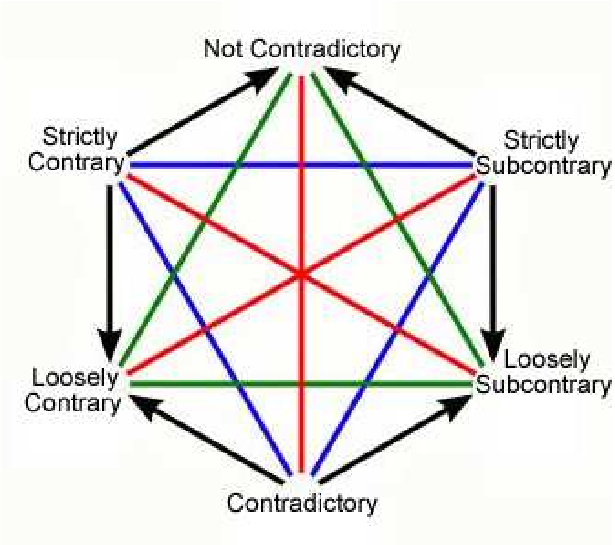
The notion of "contradiction" (*antiphasis*) was first defined by Aristotle in the fourth century BC as the opposition between an affirmative and a negative statement (*Peri Hermeneias* 6). Before that period, we had no explicit definition of a contradiction. That said, it is not impossible that Aristotle was influenced by other intellectuals or philosophers when he first described the notion of contradiction. Indeed, Aristotle attended for about twenty years the Academy of Plato and was living at a time where "new sophists" were using a new method of refutation, called the Art of contradiction (*antilogikê technê*).



The aim of the present talk is to analyse the archeology of the concept of contradiction, more precisely in Plato, and to reveal the influence that the latter had on Aristotle's reflection on contradiction and contrariety. This paper will show that it is possible to find examples of a notion of contradiction in Plato refutative dialogues, in which Socrates is described as refuting his interlocutors by demonstrating the contrary of their initial thesis. However, Plato never used the word *antiphasis* to name the act of contradicting oneself (this word seems to be the invention of Aristotle), but preferred the expression *enantia legein heautôi*, which means "to say the contrary to oneself". This expression indicates that Plato thought of contradiction as contrariety (*enantia*), a word that was already attested in Greek literature and philosophy. Moreover, in the *Apology of Socrates* (26e6-28a5), Plato described the "act of saying contrary things to oneself" as the conjunction of an affirmative statement and a negative one by giving an explicit example of a logical contradiction. A linguistic analysis shall further demonstrate that Plato indeed distinguished between true contradiction (*enantia*) and false contradiction (*antilogia*). By way of conclusion, a comparative analysis of a brief passage of Plato's *Sophist* and Aristotle's *Sophistical Refutation* confirms that Aristotle was influenced by the Platonic notion of *enantia* when he defined the notion of contradiction.

François Lepage

A Square of Opposition in Intuitionistic Logic with Strong Negation

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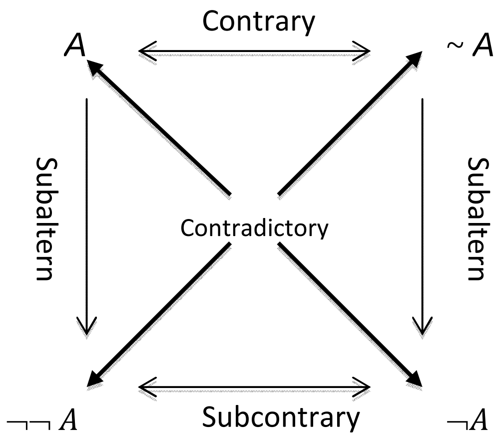
Starting with an Hilbert style axiomatization of Intuitionistic Propositional Calculus, we add axioms concerning Nelson's strong negation (~) and we obtain the following system (*IS*):

I1 $(A \to (B \to A))$
I2 $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$
I3 $A \wedge B \to A$
I4 $A \wedge B \to B$
I5 $A \to A \vee B$
I12 $\sim (A \vee B) \to \sim A \wedge \sim B$
I13 $A \wedge \sim A \to F$
I14 $\sim (A \to B) \to (A \wedge \sim B)$
I15 $(A \wedge \sim B) \to \sim (A \to B)$
I16 $\sim \neg A \to A$

I6 $B \rightarrow A \vee B$
I7 $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$
I8 $\mathrm{F} \rightarrow A$
I9 $\sim\sim A \rightarrow A$
I10 $A \rightarrow\sim\sim A$
I11 $\sim (A \wedge B) \rightarrow \sim A \vee \sim B$
I17 $A \rightarrow\sim \neg A$
I18 $\sim A \rightarrow (A \rightarrow B)$
I19 $A \rightarrow (B \rightarrow (A \wedge B))$
I20 $\sim A \vee \sim B \rightarrow\sim (A \wedge B)$
I21 $\sim A \wedge \sim B \rightarrow\sim (A \vee B)$

where $\neg A$ stands for $A \rightarrow \mathrm{F}$ is the intuitionistic negation. MP is the only rule.
We can easily prove that this system is strongly complete according to Kripke's semantics. Let $< W, \leq >$ be a canonical Kripke structure and $\alpha \in W$. We write $\vDash^{\alpha} A$ if A holds at α and $\nvDash^{\alpha} A$ if not. According to this dichotomy, the following square of oppositions holds at each α:



That is, (Contrary) there is no $\alpha', \alpha \leq \alpha'$ such that $\vDash^{\alpha'} A$ and $\vDash^{\alpha'} \sim A$ but there is (possibly) an α' such that $\nvDash^{\alpha'} A$ and $\nvDash^{\alpha'} \sim A$; (Sucontrary) if $\vDash^{\alpha} \neg\neg A$ then for all $\alpha', \alpha \leq \alpha'$ $\vDash^{\alpha'} A$ or $\nvDash^{\alpha'} A$ and $\nvDash^{\alpha'} \sim A$ but if $\vDash^{\alpha'} \neg A$ then for all $\alpha', \alpha \leq \alpha'$ $\vDash^{\alpha'} \sim A$ or $\nvDash^{\alpha'} A$ and $\nvDash^{\alpha'} \sim A$, which can be both true (when $\nvDash^{\alpha'} A$ and $\nvDash^{\alpha'} \sim A$) but not both false; (Subaltern) both are trivial; (Contradictory) both are trivial.

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Valeri Lichev

The Literary Puzzling Cases and the Problem of Recognition

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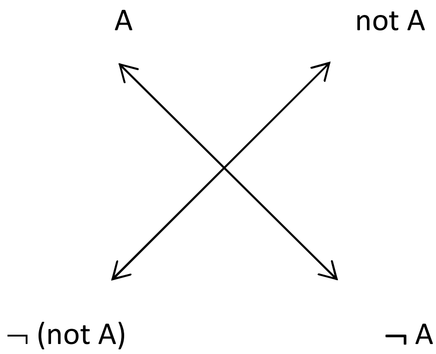
The establishment of structural linguistics in the 20th Century by F. Saussure provides a methodological basis of humanities for their turning into science. At the same time concepts

and methods from the field of science – logics, math, physics etc – commence to being received and applied into different fields of structuralism (cultural anthropology, psychoanalysis, literary criticism, philosophy etc).
An example of a formalization of the research of literary texts is the so called semiotic square (Carré sémiotique) introduced by Alguirdas J. Greimas. Using the semiotic square Greimas performs an experiment of explication of logical organization of semantic categories.
Initially the elementary structure of meaning has been understood as an oppositional relation between two articles disposed at the paradigmatic axis of language. In the second half of the 20th Century it has been found that binary relations are connected at least to two basic types: 1) **A/ Ā**, characterizing the opposition between *presence* or *absence* of a certain feature; 2) **A/ not A**, which is about manifesting of *one* and the same *feature* in *different modes*. Formally these relations can be represented in the following manner:



The relation between **A** and **¬ A** is a relation of contradiction. This is a *static definition*. From a *dynamic point of view*, the other two terms of the semiotic square **¬ A** and **¬ (not A)** arise by a *negation* of **A** and **not A**. The semiotic square is not a result of a „pure syntax“ cleared from semantic layers and this characteristic of the semiotic square makes it differ from the strict logical and mathematical constructions. Its purpose is describing the conditions of existence or generation of meanings.
The use of formal logical concepts or methods can be applied to the interpretation of the so called literary *puzzling cases* (P. Ricoeur) related to an exchange of the bodies and souls of two persons. For example, the problem of recognition as identification and acknowledgement of the character‘s personalities in Gautier‘s novel *The Transfiguration* can be described by the following table.

Reincarnation and the problem of recognition

Subjects / Object – a husband with a substituted soul	Countess (imaginary young mother)	Society (real)	Mother (real)	Count (imaginary young father)

Identification	+	–	+	–
Acknowledgement	–	+	+	–

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Vladimir Lobovikov

Squares and Hexagons for moral-evaluation-functions “Faith”, “Doubt”, “Knowledge”, “Assumption” and “Toleration” in Algebra of Formal Ethics

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The glossary for the table 1: The symbol $K^E xy$ stands for the moral-evaluation-function “*x*’s *knowing* (what, whom) *y* (in the proper *episteme* meaning of ‘*knowing’*)”. $A^D xy$ – the evaluation-function “*x*’s *assuming y (as an episteme)*”. $I^D xy$ – “*x*’s *not-assuming y (as an episteme)*”. $N^E xy$ – “*x*’s *not-knowing* (what, whom) *y*. $D^X xy$ – “*y’s* being *a doxa* of (for) *x*”. $D^M xy$ – “*y’s having a determined epistemic quality* for *x*”. These binary operations of algebra of formal ethics are defined by the following table 1.

Table 1: Moral-evaluation-functions “episteme” and “assumption”

x	*y*	$K^E xy$	$A^D xy$	$I^D xy$	$N^E xy$	$D^X xy$	$D^M xy$
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b
b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

The glossary for the table 2: The symbol $F^A xy$ stands for moral-evaluation-function “x’s *alethic* (true) *faith (not-revisable belief)* in (what, whom) *y*”. $D^N xy$ – “x’s *alethic doubt (not-removable one)* in not-*y*”. $F^N xy$ – “x’s *true faith (not-revisable belief)* in not-*y*”. $D^T xy$ – “x’s *alethic doubt (not-removable one)* in *y*”. $S^C xy$ – “x’s *alethic (true) skepticism concerning y, i.e.* x’s *alethic doubt in both: y and not-y”.* $N^S xy$ – “nonbeing of x’s *alethic skepticism concerning y”, i.e.* “*either* x’s *alethic faith* in *y*”, or “x’s *alethic faith* in not-*y*”. The moral-evaluation-functional sense of these operations is defined below by the table 2.

Table 2: Moral-evaluation-functions “alethic faith” and “alethic doubt”

x	*y*	$F^A xy$	$D^N xy$	$F^N xy$	$D^T xy$	$S^C xy$	$N^S xy$
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b

b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

The glossary for the table 3: The symbol $N^N xy$ stands for "x's *alethic* (true) *non-toleration of* not-*y*", or "x's *not-standing* (what, whom) not-*y*". $T^O xy$ – "x's *alethic toleration of y*", or "x's *standing y*". $N^O xy$ – "x's *alethic non-toleration of y*". $T^N xy$ – "x's *alethic toleration of* not-*y*". $T^C xy$ – "x's *alethic tolerance concerning y, i.e.* x's *standing both: y and not-y". $N^T xy$ – "nonbeing of x's *alethic tolerance concerning y", i.e. "either* x's *alethic non-toleration of y*", or "x's *alethic non-toleration of* not-*y*". These operations are defined by the table 3.

Table 3: Different binary moral operations "toleration" and "tolerance"

x	*y*	$N^N xy$	$T^O xy$	$N^O xy$	$T^N xy$	$T^C xy$	$N^T xy$
g	g	b	g	b	g	g	b
g	b	b	g	b	g	g	b
b	g	g	g	b	b	b	g
b	b	b	b	g	g	b	g

Gert-Jan C. Lokhorst

Alan Ross Anderson's New Square of Opposition

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In 1968, Alan Ross Anderson wrote a paper called "A new square of opposition: Eubouliatic logic" [1], in which he defined the following system of relevant eubouliatic logic: take relevant system R, add a constant G ("the good thing"), and define Rw ("it is without risk that," "it is safe that") by RwA = A → G. The eubouliatic fragment of this logic can be axiomatized as R plus axioms (A → B) → (RwB → RwA) and A → RwRwA [2]. The other eubouliatic notions can be defined in terms of Rw: HA = Rw ¬A, CA = ¬Rw ¬A, and RA = ¬RwA. Anderson read HA as "it is heedless that A," CA as "it is cautious that A," and RA as "it is risky that A," but he stressed that he was "far from satisfied with the[se] terminological choices." The relations between the four resulting eubouliatic concepts can obviously be drawn in a square of opposition. Is this logic of risk acceptable? No. There are at least three main problems.

First, the logic has theorem RwA → Rw(A & B). However, we do not normally say that if it is safe that John drinks a glass of water, then it is also safe that John drinks a glass of water and detonates a bomb. Anderson read RwA as "A guarantees that the rules are not violated," but it would be better to say that A is safe if the rules do not exclude it. The proposition that John drinks a glass of water is safe in this sense: the good thing (surviving the day, say) does not exclude it. The proposition that John drinks glass of water and detonates a bomb, on the other hand, is not safe in this sense: the good thing rules it out and it guarantees disaster. We therefore propose the following alternative analysis of safety and related notions. "It is safe that A" is to be defined as RwA = A ∘ G, where A ∘ B = ¬(A →¬B) (safety is compatibility with the good thing rather than a guarantee for the good thing). H, C and R are defined as above. The eubouliatic fragment of the new system can be axiomatized as R plus axioms (A → B) → (RwA → RwB) and Rw(A ∘ B) → (A ∘ RwB) [2].

Second, safety has to be parametrized. For example, a car may be safe for its occupants but unsafe for pedestrians. To put it even more strongly: the safer a car is for its occupants, the unsafer it is for the pedestrians who happen to be around.

Thirdly, safety is a complicated and unclear concept with many connotations. The OED lists no less than eleven different senses of "safety." Causal, epistemic, modal, probabilistic and temporal notions all seem to play some role. If we want to capture such an unclear concept in the austere language of propositional logic enriched with a propositional constant, we seem to need a logic of vagueness. However, in a logic of vagueness (for example, Lukasiewicz's three-valued logic), $A \vee \neg A$ and $\neg(A \& \neg A)$ generally do not hold, with the result that the various concepts we are considering cannot meaningfully be said to be opposed to each other and drawing a square of opposition is impossible.

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Marcos Lopes

Lexical Asymmetries in the Square of Opposition

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The graphical representation of the square of opposition is a perfectly symmetrical geometric figure. This easily invites to conceive any instantiation of it as symmetrical as well. While this may be true from an intensional perspective, it is not always the case for its extensional counterpart. This is what differentiates a finite from an infinite predicate (*praedicato finito vs. praedicato infinito*) as in "Man is just" and "Man is non-just" (Aquinas, *Exp. Lib. P. Herm.*, lib. 2, l. 2, n. 7).

Contemporary authors have attempted to extend the square of opposition as a tool to formalize cognitive and natural language oppositions as well, which in part is justified by the fact that contradiction and specially contrariety are also found in natural languages. Blanché, for one, considers that different concepts on the same semantic domain shall be structurally organized in much the same way propositions are in the square (Blanché, 1966).

If the conceptual structure of opposition is represented in natural languages by the lexicon, then contrariety is mirrored by antonymy. And although probably there would be no effective lexical means to always represent contradiction directly (without adding "not" to a primitive word), it may be inferred by mutual incompatibility of concepts, which in fact seems to have guided Blanché.

It turns out that for any pair of antonyms, one of the lexemes is always significantly more frequent in text corpora than the other, which is a correlate of the asymmetry of predicates stated above. Moreover, some preliminary tests over written language corpora indicate that not only the pair A–E is asymmetric, but so are the subcontraries I–O, and they tend to exhibit the same proportional difference that holds between A and E. As for subalterns, this proportion has been about 1/2. And, if more lexemes on the same domain are present, they will be even less frequent, with their occurrences tending to follow the proportion I/A and O/E , roughly depicting a Zipfian distribution. Finally, it follows from psycholinguistic studies that this distribution may characterize psycho-cognitive correlates (like familiarity, age of acquisition, and concreteness) of the lexemes asymmetry, so the word corresponding to A would be acquired earlier than E, and the latter earlier than I and so forth. This is yet to be investigated, but it would provide further evidence of the cognitive nature of the square of opposition.

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A Matheme Structuring the Scientific Ideosphere

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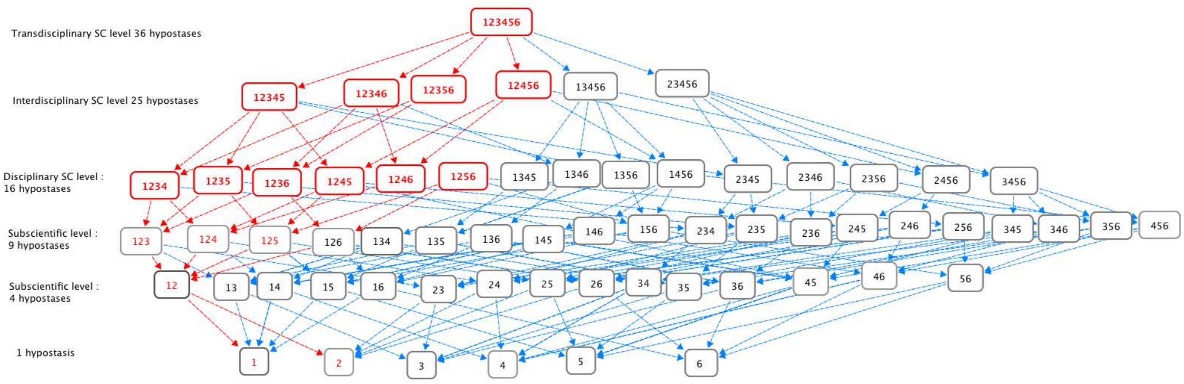
We focus on the distinction between interdisciplinary and transdisciplinary practice in science, and illustrate our approach by considering specifically computer science. The key idea is to introduce a hierarchical model where interdisciplinarity transcends the disciplines which it encompasses, and where a transdisciplinarity which transcends any interdisciplinarity comes down to being a universal practice of the debate Sciences/Citizens on a particular problem to solve.

Our model is formally based on a matheme which aims at structuring the ideosphere made up of the activity of all in the context of sciences. This is done in two steps.

First, we define the concept of hypostasis, which indicate debatable types of assumptions. Then Sciences are stressed by the hypostases associated to their forms of proof and refutation. The difference of level between scientific transdisciplinarity, interdisciplinarity and disciplinarity are related to the number of their specific hypostases.

Then we present the matheme, that is a lattice of 63 modal logics.



Each logic interprets a kind of knowledge having its specific hypostases and only 11 of these logics correspond to a scientific knowledge.

Moreover, this structure explains how transdiciplinarity is nothing more than the research of harmony and consensus in a global debate on a local query

Flavia Marcacci

An opposition triangle in the Presocratic Literature

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The Tetralogies by Antiphon are a typicall form of an antilogical contrast. The dialogical and conceptual movement focuses on the problem of fault/error and on the analysis of the key-points of the event, thus that always we have somewhat like an antilogy in a deontic context:

Prosecution: 'A' against Defense: 'non-A'.

Nobody wins and we join ‘A’ *et* ‘non-A’. And something like this seems to be in the Zeno’s fragments (especially 1, 2 and 3), and it seems to have not a *reductio ab absurdum* but just an antilogical structure:

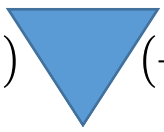
reductio ab absurdum	In the Zeno’s fragments
If *p* then *q* If ~*q* then ~*p* *p* then *q*	if ~*q* then ~*p* if ~*p* then *r et* ~*r*

In Melissus we have again a counterfactual reasoning: a special ontological “operator” is possible to identify (see Marcacci 2014) to defense an absolute monism. In the modal version it is:

$$\left(\neg \lozenge \left((p \wedge \neg q) \vee (\neg p \wedge q)\right)\right)$$

In short, in this Presocratic literature we can find an opposition triangle, with many important consequences for a correct pre-history of the Logic and of the Square of Opposition (e.g. the development of the relationship between Logic and Ontology):

Antiphon (antilogies) $\left(\lozenge \left((p \wedge \neg q) \vee (\neg p \wedge q)\right)\right)$



Melissus (Logical monism) $\left(\neg \lozenge \left((p \wedge \neg q) \vee (\neg p \wedge q)\right)\right)$

Zeno (Undetermined)

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The Structure of Ideas in the *Port Royal Logic*

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This paper addresses the degree to which the *Port Royal Logic* anticipates Boolean algebra. According to Marc Dominicy (*La Naissance de la Grammaire modern*) the best reconstruction of structure of ideas in the *Art of Thinking* is a Boolean Algebra of Carnapian properties, i.e. functions from possible worlds to extensions. Sylvain Auroux (*La Logique des Idées*) advances various reconstructions, but in the one that is formally well-defined the structure attributed to ideas is that of a non-complemented lattice. Both attribute to the *Logic* algebraic operations on ideas and extensions. To ideas they attribute an ordering relation of containment, operations of meet (idea composition) and join (idea simplification or restriction), and maximal and minimal elements. Dominicy in addition finds Boolean complementation, but Auroux argues that the *Logic*'s account of idea negation (complementation) is incoherent. Both attribute to extensions the structure of a Boolean algebra of set. Dominicy holds, but Auroux denies, that the structure of ideas is "dual" to that of extensions.

In this paper I argue that it is both exaggerated and anachronistic to read Boolean algebra into the *Port Royal Logic*. It is true that the *Logic* order ideas under a kind of containment relation and treats extensions much as we do sets. It is also true that it posits mental operations, in the medieval sense, of abstraction and restriction on ideas. The *Logic* also refers to *being* as an idea that is contained in every idea, a kind of minimal element. But it is not true that negation in the *Logic* is incoherent. It is, in fact, a variety of privative negation, a notion familiar to logic since Aristotle, with logical properties that are distinctly non-Boolean. There are many notions of duality in logic and mathematics, but the only sense that can be abstracted from the *Port Royal Logic* is the quite trivial one, also to be found in the medieval theories of concepts, that if one concept is defined in terms of another, the objects the latter signifies form a subset of those the former signifies.

The *Logic*'s account of the structure of ideas is neither mathematical nor algebraic in the modern sense. There is no attempt to define, much less axiomatize, operations on extensions or the properties of idea containment, combination or restriction. Nor is there any manipulation of formulas in deductive proofs that are justified by appeal to this structure. Rather, the *Logic* denigrates the utility of formal proof. Proofs, such as they are, are considered only as part of the syllogistic, and are explained there without appeal to the containment structure of ideas.

On the whole, the structure of ideas in the *Logic* is better understood as a more abstract version of the Tree of Porphyry, one suitable to the looser notion of "species" appropriate to Cartesianism. Though it is possible to abstract from the account a kind of lattice structure, its purpose was not to explore the formal properties of algebraic operations but rather to elaborate the psychology, ontology, and epistemology that underlie key Cartesian doctrines, like the nature of knowledge, and the ways error in the formation of ideas leads to moral vice. In epistemology, for example, any time we have a clear and distinct idea of *S* as *P*, we automatically know the proposition *every S is P.* In ethics we are lead astray as children when we form a confused idea by combining incompatible modes, e.g. when we think pleasure has a bodily cause. These explanations make appeal to the *Logic*'s theory of idea, but they are not Boolean algebra.

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Ingolf Max

What is responsible for the opposition in the square?

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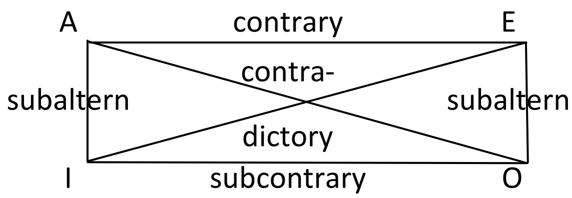
We start with the following diagrammatic form of the square:



This form is *diagrammatic* because we do not know the logical forms of the four corners A, E, I and O. Furthermore, we do not know the formal characterization of the edges (relations) between the corners. Is there any common reason to take each edge as a representative of some very general kind of *opposition*? We can add another kind of opposition: the *duality* between A and I (E and O). One of the inspiring properties of such a diagram is that each corner can be formalized in an arbitrary language of any logic we want to consider. And we can characterize the logical properties of the edges using all the possibilities of the chosen logic. Maybe we try to represent the edges by negation. Should it be only one? Or would it be nice to have one symbol for a contradictory negation and another one for contrary negation; a third one for subcontrarity and a fourth one for duality? Is subalternity a kind of negation?

A syntactically 2-dimensional framework will be presented. The 4 corners will be formalized using ordered pairs $< A, B >$ of classical formulas A and B. In this framework we can differentiate between 9 candidates of *opposition operators* ($-_n$). All of them fulfill two minimal conditions:

(1) $-_n -_n X = X$ and (2) $-_n X \neq X$ for at least one (structured) X.

Opposition operators are characterized by reduction rules within the maximal pattern $-_n < A, B > \Rightarrow < \phi AB, \psi AB >$, ϕ, and ψ are (not necessarily different) binary classical functors. In most cases we need only unary functors.
Among them we have 3 types: one *total opposition operator* ($-_1 < A, B > \Rightarrow < \neg A, \neg B >$), two *global opposition operators* (e.g., $-_1 < A, B > \Rightarrow < \neg A, B >$) and *six partial opposition operators* (among them: $-_5 < A, B > \Rightarrow < \neg A \equiv B, B >$ and $-_7 < A, B > \Rightarrow < B, A >$).

The main result is that we can reconstruct the square using astonishing different patterns:

(a) The first pattern is a more traditional one. We define the logical properties of the edges (contrarity, subcontrarity etc.) using "$-_5$" or "$-_7$" and an entailment relation "$\rightarrow$" between 2-dimensional arguments characterized by the reduction rule $< A, B > \rightarrow < C, D > \Rightarrow < A \wedge B \supset C,\ A \wedge B \supset D >$). The corners get their forms starting with A = $< \forall x(Fx \supset Gx), \exists x Fx >$. The other 3 corners of the form $< A, B >$ get the same second dimension B ($= \exists x Fx$). B is some kind of presupposition or background information about the existence of F's. We can show that every edge can be characterized by a 2-dimensional validity (including the empty domain and duality).

(b) The second pattern is in the sense astonishing that there is no overt occurrence of classical negation "$\neg$" to get an adequate propositional square. This kind of square has its roots in the system of first degree entailments. We start with an A-corner of the form $< p_1 \wedge q_1, p_2 \vee q_2 >$, use $-_7$ (simply interchanging the dimensions) and the entailment $\rightarrow_E$ with $< A, B > \rightarrow_E < C, D > \Rightarrow < A \supset C, D \supset B >$ and the same validity "$\vDash_{11}$" as in (a) defined by $\vDash_{11} < A, B > =_{df} \vDash (A \wedge B)$.

(c) We get the most astonishing formal representation using the same partial opposition operator "$-_5$" for all edges. The difference between, e.g., contrarity and subalternity, can be completely represented by well-chosen background forms "B". There are two subcases: (i) The edge-relation between any pair of corners is simply the $-_5$-negation of one corner to get the other one using for both corners the background information that their first dimensions are materially different ($\not\equiv$). E.g., $-_5 < p \wedge q, \big((p \wedge q) \not\equiv (\neg p \wedge \neg q)\big) > = < \neg p \wedge \neg q, \big((p \wedge q) \not\equiv (\neg p \wedge \neg q)\big) >$. (ii) Depending on the logical form of the corner expressions we can restrict the description of B to some internal "data": $-_5 < p \wedge q, p \equiv q > = < \neg p \wedge \neg q, p \equiv q >$, $-_5 < p \wedge q, p \not\equiv q > = < p \vee q, p \not\equiv q >$. The relation between A and I (E and O) is simply our $-_5$-negation working in both directions!

Giovanni Mion

The Square of Opposition: From Russell's Logic to Kant's Cosmology

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In this paper, I will show to what extent we can use our modern understanding of the Square of Opposition in order to make sense of Kant's double standard solution to the cosmological antinomies. Notoriously, for Kant, both theses and antitheses of the *mathematical* antinomies are false, while both theses and antitheses of the *dynamical* antinomies are true. Kantian philosophers and interpreters (including Schopenhauer, for example) have criticized Kant's solution as artificial and prejudicial. In the paper, I do not dispute such claims, but I show that our modern understanding of the Square of Opposition enables us to more naturally deliver the result Kant was aiming at. Accordingly, the paper does not pretend to be exegetically accurate. It's an attempt to revise the antinomies with the help of standard classical logic. And although such a revision entails some re-interpretation, in the end, it will actually help to unveil some of Kant's thoughts.

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The Oppositional Geometry of the Modal System K45

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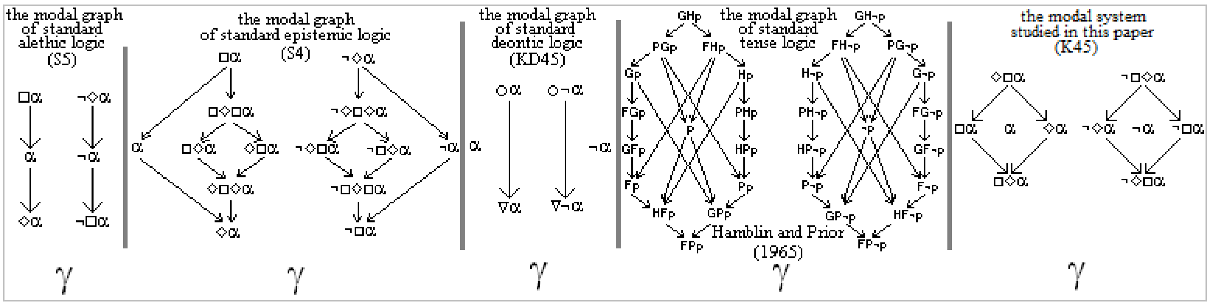
The square of opposition has known two major developments: one with the discovery, around 1950 (by Jacoby, Sesmat and Blanché), of a "logical hexagon"; and another with the discovery, in 1968 and 2008 (by Sauriol, and later by Pellissier and Smessaert), of a "logical tetrahexahedron". Since a decade such discoveries are being unified (by authors like Béziau, Guitart, Moretti, Pellissier and Smessaert) in a more or less joint framework, admitting among several names that of "oppositional geometry". In such a converging theory, useful for modelling "oppositional phenomena", at least three kinds of oppositional structures do exist: the *n*-oppositions (i.e. the oppositional bi-simplexes, the αn-structures), the *n*-closures (i.e. the βn-structures) and the oppositional generators (i.e. the γ-structures). The β-structures, generated by the γ-structures, are geometrical and finitely fractal bundles of α-structures. Previous studies have used the methodology of oppositional geometry for enquiring known systems of modal logic, since the latter can be characterised by entities (the "modal graphs", as in Chellas or in Hughes and Cresswell) which happen to correspond to what oppositional geometry theorises as γ-structures. As an example, it has been shown that S5 (the system of standard alethic modal logic) has as its oppositional backbone the $\beta 3$-structure, whereas KD45 (the system of standard deontic logic) has as its oppositional backbone the $\beta 5$-structure.



In this paper we investigate the system K45, by focussing on its modal graph, seen as a γ-structure of oppositional geometry. After having determined its corresponding β*n*-structure (i.e. its oppositional backbone), which gives us automatically the complete geometrical bundle of its α*n*-structures (i.e. its *n*-oppositions), we concentrate on the axiomatic definition of K45, which puts it, through a 3D lattice structure, into relation with ten other modal systems (among which the system S5). Thence we draw some new consequences.

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Analysis of generalized square of opposition

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In this contribution, we introduce formal theory on the basis of which we can analyze the generalized Aristotelian square of opposition (see [1]), that, in addition to the classical quantifiers, can be extended by several selected intermediate quantifiers (for example, “most”, “many”, etc.). We show that the expected relations can be well modeled in our theory (see[3]). Our analysis is motivated by Peterson's analysis as presented in his book [4], and our goal is to demonstrate that the proposed formal theory addresses well all of the problems that are encountered when considering intermediate quantifiers. It is also clear that the studied relations correspond to the proved generalized syllogisms (see[2]).

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Regaining the Square of Opposition in Formal Ontology Development

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Use of formal ontologies is becoming widespread in information systems. Forerunners of formal ontologies are scientific classification systems such as the Linnaean biological ones. Unlike biological classifications modern formal ontologies are often non-hierarchical.

A formal ontology in its basic form simply specifies all direct inclusion relationships between a finite repertoire of classes. Individuals may be conceived of as singleton classes. An assertion "P sub Q" states that class P is an immediate subclass of Q. These given relationships are often rendered as directed graphs. The subclass relationship induces a partial order relation corresponding to the relationship "all P are Q" in the square of opposition.

Accordingly, so far, formal ontologies provide only assertions of the form "all P are Q". However, it is our contention that the three other assertion forms in the square of opposition come about implicitly by appropriate, often tacitly assumed default conventions as to be explained. Assume existential import so that all classes are considered non-empty, implying that there is no empty null class.

Defaults:

1) Overlapping (i.e. non-disjoint) classes, viz. "some P are Q", has at least one common subclass.

2) Dually, classes are disjoint ("no P is Q") if they do not have a common subclass.

3) The assertion form "some P are not Q" is -- analogously to class overlap -- achieved by requiring that there be a subclass of P which is disjoint with Q. More radically this assertion may be held simply in the case that "all P are Q" does not hold.

These default rules are routinely adopted in ontology development without mentioning. Appealing to these conventions, the 4 sentence forms in the square are effectively made at disposal. We discuss a first order metalogical formalization of the 4 sentence forms with classes reified as individual constants elucidating the logical relationships between the sentence forms.

Our formalization appeals to non-provability. Non-provability incurs non-monotonicity, implying that extension of an ontology with additional subclass relationships may call for retraction of derived square of opposition relationships. This reflects the crucial distinction between the closed world assumption (CWA) and the open world assumption (OWA).

Maria Fernanda Niño

The diplomatic virtual presence in the light of the semiotic square

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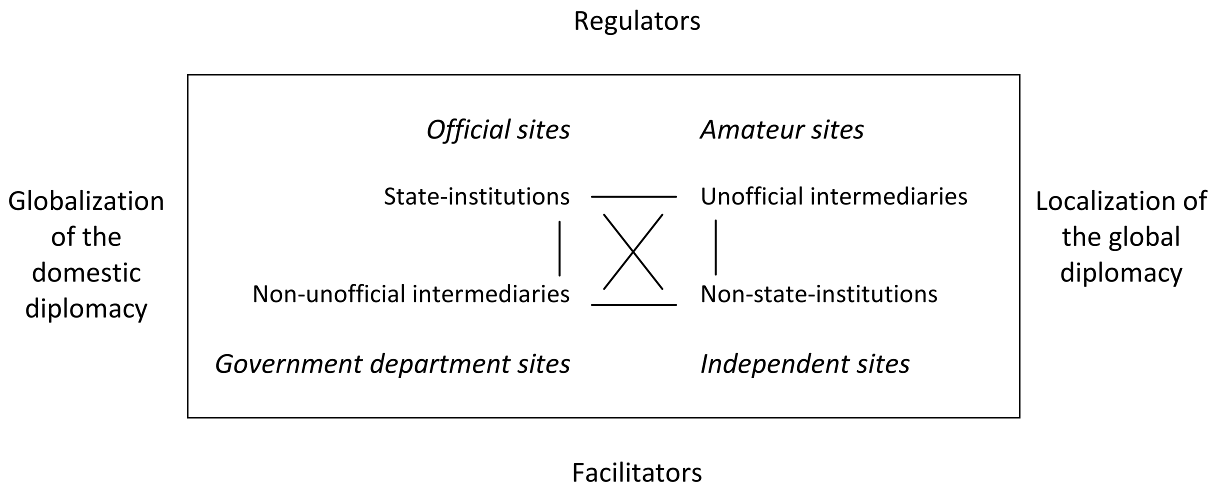
New technologies have led many local and national governments across the world to invest in new ways to approach and to engage citizens as well as to increase transparency and accountability. Those transformations have also had an impact on the diplomatic affaires, where embassies try to respond both to digital diplomacy and e-government initiatives.

This paper contains an overview of the diplomatic virtual presence by focusing on the conceptual and practical contributions of the semiotic square. From the semiotic analysis perspective, the semiotic square is a tool used to map logical relations of a semantic category. Therefore, the case study that is conducted here not only demonstrates the applicability of the semiotic square to map the meanings that are invested on the embassies websites but also, the complexity of the conceptual and the visual representation that emerges from hierarchizing the basic four –terms of the structure, what allows setting out the superior levels of the online diplomatic use, misuse and non-use.

Diplomatic virtual presence



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Square for Logical Values

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This abstract proposes representation of logical values as pairs of sets, which operations of set algebra: intersection and union correspond to logical operations: conjunction and disjunction.

J. Dunn defined **T**, **F**, **N** and **B** as {true}, {false}, {}, {true, false} respectively.

Tables for conjunction and disjunction:

$\wedge$	T	F	N	B
T	T	F	N	B
F	F	F	F	F
N	N	F	N	F
B	B	F	F	B

$\vee$	T	F	N	B
T	T	T	T	T
F	T	F	N	B
N	T	N	N	T
B	T	B	T	B

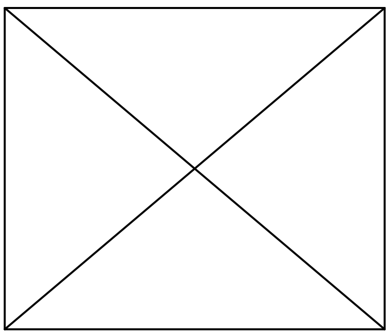
R.Muskens notes that intersection and union of these sets don't correspond to conjunction and disjunction.

So other definitions of logical values are introduced.

Set of subsets from one-element set {truth} was created $P(\{truth\}) = \{\{truth\}, \varnothing\}$.

And we have square for 4 values, which T-square (shortly T_4) called.

N = $\langle\varnothing, \{truth\}\rangle$ T = $\langle\{truth\}, \{truth\}\rangle$



F = $\langle\varnothing, \varnothing\rangle$ B = $\langle\{truth\}, \varnothing\rangle$

Note that all values are in opposition to each other.

Next definition is introduced:

$$S(A) = \begin{cases} \langle\{truth\}, \{truth\}\rangle \text{ if } A \text{ is true and } A \text{ isn't false,} \\ \langle\varnothing, \varnothing\rangle \text{ if } A \text{ is false and } A \text{ isn't true,} \\ \\ \langle\varnothing, \{truth\}\rangle \text{ if } A \text{ isn't true and } A \text{ isn't false,} \\ \langle\{truth\}, \varnothing\rangle \text{ if } A \text{ is true and } A \text{ is false.} \end{cases}$$

We have the relations in two-dimensional space of this square T_4:

$(S(A) \cap S(B)) = S(A \wedge B)$,

$(S(A) \cup S(B)) = S(A \vee B)$.

Now intersection and union correspond to conjunction and disjunction.

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Ana Petrache

The use of paraconsistent negation and of paracomplete negation in the theological discourse

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The aim of my presentation will be to discuss the implications for the theological discourse of the paraconsistent contradiction and of the paracomplete contradiction. Starting from the observation that paraconsistent logic it is more plausible to be use in theology that intuitionist logic, because the ontological argument can be prof only in S5, and that the intuitionist logic reject the validity of the argument, without LEM the reduction to the absurd became impossible, I want to discuss the opportunity that paraconsistent star of opposition and paracomplete star of opposition can offer to the theological discourse.

My aim is not in logic it-self but in the use that theologians can make with these new discoveries. Precisely I like to research what are the implications of the various forms of the square of opposition for the ontological argument, how various formalization of non-contingency interfere with the ontological argument and how various formalization of contingent interfere with the conception of an interventionist God. In the end, I would like to discuss why theology situates it-self between the necessity of God existence and the contingency of his action in history.

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Probabilistic Interpretations of the Square of Opposition

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We investigate the square of opposition from a probabilistic point of view. Probability allows for dealing with exceptions and uncertainty. We will interpret the corners of the square by means of (precise or imprecise) conditional probability assessments. They will be defined within the framework of *coherence*, which originally goes back to de Finetti. In this framework probabilities are conceived as degrees of belief, where conditional probability is defined as a primitive concept. Coherence allows for dealing with partial and imprecise assessments. Moreover, the coherence approach is especially suitable for dealing with zero antecedent probabilities (i.e., here conditioning events may have probability zero): This is relevant for studying different probabilistic interpretations of the *existential import*.

In this talk, we will discuss probabilistic notions of the existential import and present probabilistic interpretations of *universally affirmative* and *negative* as well as *particular affirmative* and *negative* propositions. After choosing appropriate probabilistic constraints for defining the four basic types of propositions and the existential import, we will present a probabilistic version of the traditional square of opposition. We will discuss in what sense the traditional relations—*contradictories, contraries, sub-contraries*, and *sub-alternations*—are also contained in the probabilistic square of opposition. Moreover, we will generalize our probabilistic interpretation of the basic syllogistic concepts to construct probabilistic versions of selected syllogisms. We will also relate them to inference rules in *nonmonotonic reasoning*. Finally, we will discuss how probabilistic syllogisms could serve as a rationality framework for human reasoning about quantifiers within the so-called "*new psychology of reasoning"*.

Claudio Pizzi

Composition and generalization of modal squares of oppositions

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The first point treated in the paper aims at showing that the basic Aristotelian modal square of oppositions may be seen as special case of an Aristotelian cube, i.e. of a particular combination of six squares, two of which are Aristotelian and four of which are "semiaristotelian" in a suitably defined sense. It is remarkable, anyway, that an Aristotelian square is always such with respect to a reference modal system S, so that it may happen that inside the same cube two squares may be written in different languages.

An Aristotelian cube is a degenerate cube when the two Aristotelian squares composing it are equivalent with respect to the given reference system S, while an Aristotelian square is a degenerate square when subalternants propositions are equivalent with respect to the the S. Blanchè's Hexagon may be seen as a partially degenerate cube.

A composition of squares is defined as a composition of the corresponding vertices of the two squares (i.e. as a conjunction of the upper vertices and a disjunction at the lower). It may involve squares in standard position but also "rotations" of them. The properties of proper and improper composition are an object of an investigation to be performed with the tools of modal logic. In the second parte of the paper it is claimed that an Aristotelian square may be seen not only as the special case of an Aristotelian cube, but also the special case of a sub-Aristotelian square, i.e. of an Aristotliean square whose interrelations depend on some specific extralogic assumption. It is argued that the operation of composition of squares as above defined may be extended in a natural way both to subaristotelian squares and to Aristotelian cubes.

Jiří Raclavský

Semantic Square of Hyperintensional Semantics, Logic and Identity Statements

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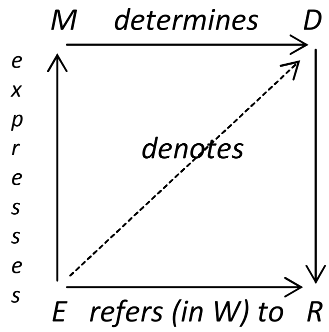
(*Part I. Approaching the Square*) Identity statements led Frege and Church to several important semantic questions and answers. Carnap, Montague 1974 and others explained Frege's famous *semantic triangle* "expression *E* – meaning *M* – denotatum *D*" as deploying *possible world intensions* as *M*s, while extensions are their values in a given possible world(s) *W*. A weakness of *intensional semantics* induced a seeking of *hyperintensionally* individuated meanings (*cf*. e.g. Lewis 1970, Cresswell 1985; my background hyperintensional theory is Tichý 1988) whereas a hyperintension would be inserted between *E* and an intension.
(*Part II. Describing the Square*) We get the *semantic square*:



A structured complex *M*, the hyperintension, determines (in an algorithmic way) a certain intension *D* whose value at a given *W* is the referent *R*. Two such meanings M_1 and M_2 can be equivalent, i.e. determining the same *D*, without being identical. Each *M* is thus specified by i. the object *O* it determines and ii. the way how it determines *O*.
(*Part III. Metaphysics of the Square: Weak Oppositions*) The oppositions manifested in the Square naturally differ from those present in the Logical Square of Oppositions, nevertheless they exist. We will discuss them in the talk.
(*Part IV. Application of the Square to Semantics*) Obviously, two *E*s can have the same *R* without having the same *D*; two *E*s can have the same *D* without having the same *M*. This gives us a number of possible interpretations of the identity statements E_1=E_2 (e.g. 'the Morning star = the Evening star' concerns an identity of R). The classical *Frege's Puzzle* is thus resolved (the New Frege's puzzle is not, *cf*. 'Tullius=Cicero', 'London=Londres', etc.).

(*Part V. Logic of the Square and Identity Statements*) An admission of intensions as values for *x* in the *logical form x*=*x* of the identity statements leads to the failure of classical logical laws such as Substitutivity of Identicals and Existential Generalization. An attempt to fix the rules should cover also the possibility of hyperintensions as values for *x*.

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Stephen Read

Aristotle's Cube of Opposition

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In *Prior Analytics* I 46, Aristotle discusses negation extensively, distinguishing 'isn't *F* ' from 'is not-*F* ', the latter an affirmative predication while the former is a negative one. He claims that the affirmation does not follow from the negative one, though the converse entailment does hold. As he had remarked in the *Categories* (13b29-33), "if Socrates does not exist, . . . 'He is ill' is false but 'He is not ill' is true." But 'is ill' means 'is unwell', so 'He is not ill' does not entail 'He is unwell'.

Thus, given a single statement '*A* is *B*', we can form its opposite in two ways, placing the negation on the copula or on the predicate: '*A* isn't *B*' or '*A* is not-*B*'. The first forms the contradictory of '*A* is *B*', the latter its contrary: '*A* is *B*' and '*A* is not-*B*' can both be false—if Socrates does not exist, 'Socrates is well' and 'Socrates is unwell' are both false.

Turning from singular statements to general adds a further possibility for formation of an opposite. For example, 'Every *A* is *B*' admits three opposites, formed by placing the negation before the determiner of the subject, before the copula or before the predicate: 'Not every *A* is *B*', 'Every *A* isn't *B*' and 'Every *A* is not-*B*'. These statements in turn admits opposites by further placing of negation, forming eight statements in total. We can set them out in the form of a Cube of Opposition.

By tracing out the logic of these eight statements, we can show that Aristotle placed no requirement that the terms be non-empty. In fact, existential commitment goes with quality, not quantity, thus satisfying all the demands of Apuleius' Square of Opposition.

Christian Retoré

Aristotle's square of opposition and Hilbert's epsilon: some linguistic remarks

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The square of opposition can be revisited according to two convergent views on quantification: the generic interpretation of noun phrases and Hilbert's operators ε,τ. As

Slater observed, the term logic of Aristotle can be properly expressed with Hilbert's ε and τ operators. The ε operator builds a term *εx.F(x)* from a formula *F* — the variable *x* is bound in *F* by *εx.* The deduction rules are the usual ones for quantification: from *F(a)* infer *F(εx.F(x))* and from *F(x)* without any free occurrence of *x* in any hypothesis, infer *F(εx.¬F(x))* — remember that *εx.¬F(x)* is *τx.F(x),* the generic element used in mathematical proofs of universal statements. One then has *F(εx.F(x))≡∃x.F(x)* and *F(τx.F(x))≡∀x.F(x):* ε-formulae can properly express quantification, even without the usual quantifiers.
Linguistically, *ε* terms and formulae should be preferred to generalized quantifiers (GQ):

1. ε-terms can interpret a single quantified noun phrase without a main predicate, e.g. *"a student",* while GQ cannot.
2. ε-formulae follow the syntactic structure while GQ don't: "*Bach composed a mass"* is closer to *composed(Bach,εx.mass(x))* than to *∃(λx.mass(x)) (λx.composed(Bach,x)).*
3. ε-terms and formulae respect the linguistic asymmetry between the two predicates of an **I** sentence. GQ wrongly assign the same logical form to "*Some politicians are crooks*" (attested) and to "*Some crooks are politicians*" (unlikely), while their respective ε formulae differ. For this reason, we prefer the original formulation of the **O** sentences, namely *"not every X is Y",* to its modern formulation *"some X are not Y".*

The beautiful symmetries of Aristotle's square can be rephrased with ε and τ, but the loose use of ε-terms in linguistics is much more appealing. Indeed, an **I** sentence like *"some X are Y"* whose usual logical form is *K= ∃z.(X(z)&Y(z))= X(e)&Y(e) with e=εz.(X(z)&Y(z))* can also be interpreted as *L=Y(εz.X(z)). The formula L&X(εz.X(z))* is equivalent to *K=∃z.(X(z)&Y(z))* but L itself is not equivalent to any first order formula! The **A**, **E**, and **O** quantified formulae can also be given an L-style logical form, yielding different but interesting symmetries in the square of opposition. In addition, ε-terms can also handle sentences with multiple quantifiers as first order logic does but also as the L-style interpretation suggests: that way, *ε*-formulae and terms provide a rather elegant account of under-specification.

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Serge Robert and Janie Brisson

Klein Groups as Squares of Oppositions and the Explanation of Fallacies in Reasoning

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Piaget (1949) has shown how classical propositional logic contains many Klein groups. These groups are structures that can be considered as squares of oppositions: instead of oppositions between atomic statements, they establish oppositions between binary connectives on propositions. Such structures contain, relative to a connective, the following operators: the identical operator on this connective (I), its inverse (N), its reciprocal (R) and its dual (D, named "correlative" by Piaget). What makes these groups squares of oppositions is that, beginning with a given operator (which is its own identical (I)), its N-operator is its contradictory operator, its R-operator is its contrary or subcontrary operator and there is also a subaltern relation between the given operator and its dual. Some Klein groups in propositional logic are genuine ones, in the sense that they contain distinct I, N, R and D operators. This is the case with the group of conditionals (P ⊃ Q, Q ⊃ P, ~P&Q and P&~Q) and the group of disjunctions (P ∨ Q, P | Q, P&Q, ~P&~Q). On the other hand, other groups are simplified or crushed groups, such that they form opposition segments instead of squares. This happens when I=D and R=N, or when I=R and D=N, which is the case between the biconditional and the exclusive disjunction.

Many experiments in cognitive science in the last fifty years have shown that adults have a spontaneous tendency to make systematic fallacies in logical reasoning. It has been observed that people treat conditionals as if they were biconditionals, while other studies show that inclusive disjunctions are treated as being exclusive. So, in conditional reasoning, people tend to accept as valid not only the *modus ponendo ponens* and the *modus tollendo tollens*, but also the conditional fallacies, i.e. the affirmation of the consequent and the negation of the antecedent. In the case of disjunctive arguments, they accept not only the *modus tollendo ponens* as valid, but also the fallacious *modus ponendo tollens*.

Our thesis is that we can model logical reasoning (valid and fallacious) as a dynamic between genuine and crushed Klein groups. Consequently, the fallacies can be seen as an oversimplification of the information given in the premises, treating the squares of oppositions as segments of oppositions, thus neglecting distinct dual operators. This way, many fallacies have common structural properties based on a misuse of the square relations at work between different propositional connectives. Following Marr's theory of different levels of explanation, our model is computational and not algorithmic: far from pretending that naive reasoners mentally manipulate Klein groups, we rather suggest that valid and invalid reasoning can be modeled using a group-based normative theory, that is, Klein groups. There are at least four advantages with our thesis. Contrary to the current literature, 1) it provides an explanation of classical reasoning at a computational level and 2) it makes predictions on fallacies that might be made with other logical connectives (like fallacies on the *modus tollendo ponens* with incompatibilities), on which no experiments have been made. Moreover, 3) it can provide AI with a unified theory for the modeling of human deductive reasoning and 4) finally, it gives clues for the development of pedagogical strategies for the improvement of reasoning, through an awareness of distinct duals.

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Aristotle's Modal Quadrant and Two-Sided Modalities

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The main aim of the paper is to explore the Aristotelian Quadrant (De Int., 13) as a possible way of representing geometrically the logical relations of oppositions between modal propositions, as an alternative or extension of the Square of Modal Oppositions.

I	II
Possible to be	Not possible to be
Not impossible to be	Impossible to be
Not necessary to be	Necessary not to be
III	IV
Possible not to be	Not possible not to be
Not impossible not to be	Impossible not to be
Not necessary not to be	Necessary to be

As is well known, this quadrant is later rejected as an incorrect representation of modal oppositions in the course of chapter 13.
We first give a reconstruction of Aristotle's reasoning leading to this rejection, with special interest to its reliance on geometrical properties of the Quadrant, namely the fact that, all the modalities on the same horizontal line are supposed to be contradictories, and that vertical translation should preserve contradictoriness of pairs of modalities.
Then we show how this correspondence between geometrical and logical properties is affected by the introduction of two-sided modalities. For it has been shown (Khamara, ms.) that Aristotle's initial Quadrant is indeed correct if all the modalities are interpreted as two-sided.
The upshot of the discussion is that Quadrants prove useful to reason about the opposition of modal propositions and represent relations which cannot be pictured within the more famous Square of Modal Oppositions.

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David H. Sanford

Necessity, Extremity, and Identity: Three Problems about Contraries

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I. Corresponding statements of the forms “All F’s are G’s” and “No F’s are G’s” are contraries.

II. If two statements are contraries, although they cannot both be true, they can both be false.

I. and II. are in conflict. Some statements of the form “All F’s are G’s” (and also some of the form “No F’s are G’s”) are necessarily true. If one of a pair of statements is necessarily true, then both cannot be false. Humberstone (2003) argues that the ways Sanford (1968) suggests to evade this conflict are all inadequate.

Humberstone resolves the conflict in another way: the contrary relation holds primarily not between statements but between forms of argument. So long as something satisfies the subject term, the argument form “All F’s are G’s, therefore it is not the case that no F’s are G’s” is valid. As with any valid form of argument, every instance of it is a valid argument. On the other hand, the argument form “It is not the case that all F’s are G’s, therefore no F’s are G’s” is invalid. As with any invalid form of argument, not every instance of it is a valid argument.This is not to say that every instance of it is an invalid argument.

Such definitions in terms of forms of argument, and also previous treatments, fail to account for the view that contraries are opposite extremes. An initially appealing formal amendment doesn’t work. Until we can express extremity formally, it is best to specify in advance the forms of statements under consideration.

Mereological identity fits nicely in a square of opposition. Having every part in common (identity) is contrary to having no part in common (disjointness). It is not obvious, however, that all identity is mereological. When we consider the identity relation as it appears in predicate logic with identity, it is difficult to see how any contrary to an identity statement can be true. Thus two questions remains unanswered. Is this a hitherto unappreciated feature of identity that deserves further study? Or is there another kind of definition that allows the truth of the contrary of an identity statement?

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Denis I. Saveliev

On paradoxical and non-paradoxical systems of propositions referring to each other

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As well-known, all classical paradoxes involve a kind of self-reference. A paradox without any explicit self-reference was proposed by Yablo twenty years ago in [1] (for a subsequent discussion see [2]—[6]). This new paradox can be considered as an unfolding of the paradigmatic Liar Paradox: it consists of propositions indexed by natural numbers such that each of the propositions states «all propositions with greater indices are wrong». Our purpose is to investigate arbitrary systems of propositions some of which state that some others are wrong, and to learn which of these systems are paradoxical and which are not. For this, we introduce a first-order theory in a language with one unary and one binary predicates, *T* and *U*, consisting of two axioms:

$\forall xy\ (Tx \rightarrow (Uxy \rightarrow \neg Ty))$;
$\forall x\ (\neg Tx \wedge \exists y\ Uxy \rightarrow \exists y\ (Uxy \wedge Ty))$.

Intuitively, variables mean propositions, *Tx* means «*x* is true», and *Uxy* means «*x* states that *y* is wrong». A model *(X,U)* is non-paradoxical iff it can be enriched to some model *(X,T,U)* of this theory, and paradoxical otherwise. E.g. a model of the Liar Paradox consists of one reflexive point, a model of the Yablo Paradox is isomorphic to natural numbers with their usual ordering, and both are paradoxical. Generalizing these two instances, we note that any model with a transitive *U* without maximal elements is paradoxical. On the other hand, any model with a well-founded U^{-1} is not. We provide a classification of non-paradoxical models.

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Language and Modal Logic of Strategic Decisions

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Background

Historically, the square of opposition has been developed by legal scholars as a reasoning scheme to analyze arguments and positions. Arguments are n-dimensional. They consist of rational and formal, social and rhetorical dimensions. Arguments are used to express opinions and, furthermore, to convince other agents to agree with a certain viewpoint and/or to join a specific world view or system. A vivid part of arguments are strategies or a specific canonical structure. But, strategies are just action briefings based on goals and concrete instructions. Strategies are a fundamental part of any kind of management doctrine and an important aptitude in legal practice. Although, strategies and arguments are well investigated subjects in logic, legal theory, and management sciences, there has been no investigations in, firstly, the logical or modal logical structure, i.e. the modals used in strategies and, secondly, in the limits of any kind of logic based on such modals, so far.

Goal of our presentation

Our presentation aims at introducing the modals associated with the language and logic of strategic decisions, its origin in the square of oppositions, and providing an outlook on the meta-theory expressing propositions on the language and logic of strategic decisions.

Fabien Schang

Silence has no logical value. Making sense of nonsense, behind and beyond the Tractatus 7

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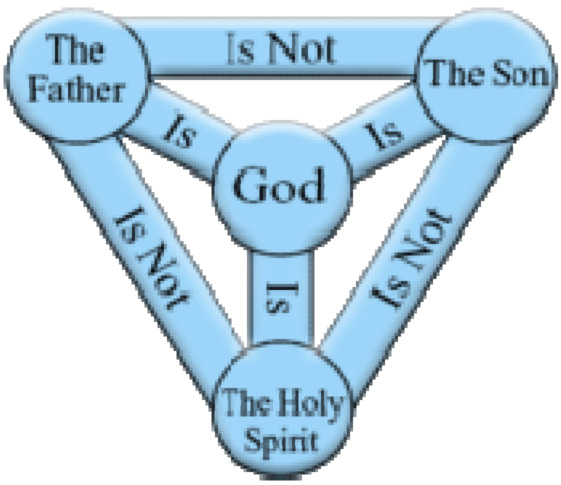
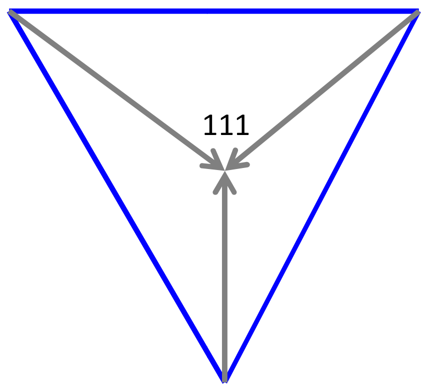
Any self-respectable philosopher knows these last words from Wittgenstein's *Tractatus*: *Wovon man nicht sprechen kann, darüber muß man schweigen*.

Now what exactly cannot be talked about? Everything mystical or transcendental, Wittgenstein would have said. Is God a mere contradictory object, or is He something more than a logical falsity? To handle such problems ist the purpose of the paper, by means of a more fine-grained theory of opposition.

A general framework is proposed to streamline the whole debate in a formal semantics: Question-Answer Semantics (thereafter: **QAS**). Three main features of **QAS** may throw some further light on the following religious triangle of contraries:





The point is that the question about transcendental entities should not be: is God a self-contradictory object? But, rather: is it anything at all? The confusion between antilogy, contradiction and nothingness is taken to create some confusion in such a discourse about transcendence. A clarification about what the core concept of *negation* means (whether exclusive or inclusive) is in order, accordingly.

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Marcin J. Schroeder

From Negation to Infinity: Logic for Non-trivial Integrated Information Systems Cannot Be Finite

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Information in its general conceptual framework of the one-many opposition has been formalized by the author in his earlier publications in terms of closure spaces. While information is identified there with a filter in the complete lattice of closed subsets, the lattice of closed subsets is playing a role of the logic of information (Schroeder, 2012). In the restriction to the linguistic forms of information, as for instance in the traditional propositional logic, this lattice is Boolean, and therefore it is completely reducible to the direct product of trivial two-element Boolean lattices. In a more general case, the logic of information systems may be irreducible, partially or completely which reflects the level of information integration (Schroeder, 2009). Completely irreducible are for instance quantum logics and logics of geometric systems.

Present paper is starting from the important, but not frequently recollected result of Ivert and Sjödin (1978) showing that nontrivial quantum logics cannot be finite. By similar mathematical reasoning, it can be shown that fully integrated information, or even partially integrated one but not reducible to trivial components, requires infinite logic (i.e. infinite lattice of closed subsets in the closure space describing information system). The proof is based on the relation between closed subsets which can be associated with the algebraic representation of syllogistic and the Square of Opposition (Schroeder, 2012).

Since negation in the logic of information has formal properties identical with the geometric orthogonal complementation, it is not a surprise that historically the first occurrence of the need for actual infinity occurred with the irrational numbers, i.e. numbers which could not be presented as a result of a finite-step process consisting of simple manipulations of units, in the context of geometry, i.e. geometric information systems, and specifically in the context of orthogonality (the length of the diagonal in a square).

Infinity of the logic for integrated information systems has the fundamental importance for the issue of designing computational systems involving information integration. For instance, as a consequence, no Turing machine can implement the mechanism of information integration, since it requires simultaneous manipulation of the infinite number of states of the system, and the number of the states is necessarily infinite due to the infinity of the logic of information. On the other hand, association of information integration with consciousness suggests that the cognitive mechanisms of the brain require infinite number of states as well. This in turn can be used as an argument for the necessity of new models of computation which could describe cognitive functions of the brain and of consciousness.

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Pieter A.M. Seuren

Metalogical hexagons in natural logic

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It is possible to construct metahexagons for logical relations (Proper) Entailment, Equivalence, (Proper) Contrariety, (Proper) Subcontrariety and Contradictoriness. The construction of such metahexagons shows that, when restricted to contingent propositions, separate complete hexagons come about for two classes of logical relations, a class of opposition relations, consisting of (Proper) Contrariety, (Proper) Subcontrariety and Contradiction, and a class of entailment relations, consisting of (Proper) Entailment and Equivalence. The paper shows in detail how these hexagons are constructed and what the possible consequences are for natural logic.

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Incommensurability of the Squares

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The idea that there exist logical relationships between pairs of propositions, when they have same subject and predicate terms is developed in Apuleius of Maduara's commentary on The Perihermaneias[4]. Boethius used a geometrical figure to express relations among propositions based on the doctrines of Aristotle. This came to be known as the Traditional Square of Opposition. Certain controversies surrounded the Square, from the beginning [7], though it was defended [5]. The task to modify it was then left to modern logicians. The modifications by moderns led the relations of contrary, sub-contrary and subaltern fell apart, while the relation of contradiction subsists. The cross of opposition [6] which survived is called the Modern Square.

I intend to survey the history of Squares outlining the differences in Aristotle along with medieval advancements [2] and modern interpretations. My next task is to show revised square as a further progression, of developments that took place in the nineteenth-twentieth century. With these, I argue that the traditional and modern squares are incommensurable using primarily Feyerabend [1] and Kuhn's [3] interpretations. Feyerabend opines, “a theory is incommensurable with another if its ontological consequences are incompatible with the ontological consequences of the latter”. Similarly, Kuhn argues, “What differentiated these various [opposing] schools was not one or another failure of method – they were all `scientific' – but what we shall come to call their incommensurable ways of seeing the world and of practicing science in it”. In this talk, we take various readings (versions) of incommensurability and map it with both the Squares.

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Fabrice Silpa

Square of Opposition in Egyptology. A logical study of Maat

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We propose in this paper a study of the Ancient Egypt ideologic system contained in Maat thanks to the Square of Opposition. Indeed, after a study of the most popular tales of Ancient Egypt, we consider the logical system of justice symbolised by Maat.

Maat is the goddess of justice but it is also all the Egyptian civilization (J. Assmann, 1999) and the center of a sophisticated ideological system (B. Menu, 2006). Our work shows that this system can be analysed in the Square of Opposition but also into more complex geometrical objects.

Marcos Silva

What does it mean to negate a proposition?

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Here we compare the logical behavior of negation in Wittgenstein´s *Tractatus* (1921) with Demos´ account of denial (1917). Both i) reveal a Russellian suspicion about superficial grammar which justifies logical analysis and ii) reject any account of negation which demands the existence of negative facts. As a result of this rejection, we diagnose in both authors an unexpected mandatory emergence of various systems of propositions in order to deal with fine oppositions and exclusions. This is clear, for example, in some texts of Wittgenstein return to philosophy in 1929 and in Demos´ account of negative proposition as always describing a contrary positive proposition. In this respect, even if we hold negation as a pure syntactical device, at least in some context of interpretation, it shall bring a handful of complex semantic information, potentially infinite (eg. in the ascription of degrees to empirical qualities or of color to visual points). This shows *inter alia* that the application of negation may presuppose highly organized domains with several fine inferences and exclusions.

Take, for instance, a teacher who asks his student to draw at the board the fact that a black cat is not on a large table. After some moments of introspection, the puzzled student accepts the challenge, goes to the board and draws a modest black cat besides a large table. His teacher rejects peremptorily this drawing stating he never asked the student to draw a black cat besides a large table, but that it is not the case that the black cat is on a large table. He goes himself to the board, erases the first drawing done by his disappointed student and draws an imposing black cat on a large table and then makes a great "x" over the whole drawing. "You see!" says triumphantly the proud teacher. "That is the drawing of the fact that the black cat is *not* on the large table!"

This paper will defend that the teacher´s approach to the matter is mainly a tractarian one: negation should not be a part of the world (*Bild*, picture, drawing) and it should be "put over" the whole proposition (*Bild*, picture, drawing) meaning that the whole described situation is excluded. This implies that *either* the drawing with the great X over it *or* the drawing without it must be the correct one, without a third alternative. Moreover, we will also defend that the student has no reason to be sad. He is not wrong. He is intuitively using a finer way of negating or excluding a situation, by opposing it to another one or, in other words, by showing a contrary situation, since either drawings or pictures cannot be true together, but false together. Whereas his teacher´s drawing accepts just two (exhaustive and exclusive) alternatives, the student´s drawing represents just one alternative of a great multitude of alternatives. For instance, take cat being behind the table or under the table, on the left of it, on the right of it, etc. This is crucial for our discussion: the teacher´s and the student´s drawings are indeed excluding the fact that the black cat is on the large table, but *differently*. This difference is central to understand the role of negation in some different context.

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Hans Smessaert

The Logical Geometry of the Rhombic Dodecahedron of Oppositions

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The internal structure of the (strong) Jacoby-Sesmat-Blanché hexagon (JSB) has been exhaustively characterized in terms of the 3 Aristotelian squares that can be embedded into it. Similarly, some initial results have been obtained concerning the internal structure of the rhombic dodecahedron (RDH) of oppositions: 6 strong and 4 weak JSB hexagons can be embedded into an RDH. However, because of the greater complexity of the RDH as compared to the JSB, an exhaustive analysis of its internal structure has been lacking so far. The main aim of this paper is to describe the tools and techniques we have used to obtain exactly such an analysis. This involves examining larger substructures (not just squares, but also hexagons, octagons, etc.), distinguishing various families of substructures (strong JSB, weak JSB, etc.) and establishing the exhaustiveness of the typology.

An RDH is a three-dimensional object with 12 rhombic faces and 14 vertices. Since an RDH is dual to the cuboctahedron, it consists of a cube (8 vertices) and an octahedron (6 vertices). An RDH has been used to visualize the Aristotelian relations between expressions in various logical systems (e.g. classical propositional, first-order, modal and public announcement logic) and lexical fields (e.g. comparison, colour terms, set inclusion, subjective quantification). These expressions are encoded by means of bitstrings of length 4; those of levels 1 and 3 (e.g. 1000 and 1110) occupy the 8 cube vertices, whereas those of level 2 (e.g. 1010) occupy the 6 octahedron vertices. Furthermore, the contradiction relation is visualized using the central symmetry of the RDH: contradictory bitstrings (e.g. 1100 and 0011) occupy diametrically opposed vertices at a maximal (Euclidean) distance from one another.

The key notion in describing the internal structure of the RDH is that of a *σ-structure*. A σ_n-structure consists of *n* pairs of contradictory bitstrings (PCD), and is visualized by means of a centrally symmetrical diagram; e.g. a classical Aristotelian square and a JSB hexagon are a σ_2- and a σ_3-structure, respectively. From the σ-perspective the RDH is a σ_7-structure, which already allows a rough description of its internal structure; e.g. the number of squares (σ_2) inside the RDH (σ_7) can be calculated as the number of combinations of 2 PCDs out of 7: $\binom{7}{2} = \frac{7!}{2!(7-2)!} = 21$. This combinatorial technique not only recovers well-known results (e.g. the number of squares (σ_2) inside a hexagon (σ_3) is $\binom{3}{2} = 3$), but also yields new results: the number of hexagons (σ_3) inside the RDH (σ_7) is $\binom{7}{3} = 35$. Among these 35, 10 were already known (viz. the 6 strong and 4 weak JSBs), but the remaining 25 are largely unknown, except that they must include at least some of the Sherwood-Czezowski (SC) family.

The next step is to construct a principled typology of the various families of σ-structures inside the RDH. Recalling that the RDH (σ_7) fundamentally consists of a cube (**C**; σ_4) and an octahedron (**O**; σ_3), each σ_n can not only be seen as a combination of *n* out of the 7 PCDs of the RDH, but also in more detail as a combination of *k* out of the 4 PCDs of **C** and *m* out of the 3 PCDs of **O** (for *k* + *m* = *n*). The number of $\mathbf{C}_k\mathbf{O}_l$-structures is $\binom{4}{k}\binom{3}{m}$. A further subdivision of certain families requires augmenting the **CO**-perspective with a perspective based on *isomorphisms* of Aristotelian diagrams. Both perspectives are independent of each other, in the sense that there are families that can be distinguished by the former but not the latter, and vice versa. We will finish the presentation by defining a fundamental complementarity between 'small' and 'large' σ-structures inside the RDH (symbolically: $|\mathbf{C}_k\mathbf{O}_m| = |\mathbf{C}_{4\text{-}k}\mathbf{O}_{3\text{-}m}|$).

Hans Smessaert & Lorenz Demey

The Unreasonable Effectiveness of Bitstrings in Logical Geometry

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Logical geometry (LG) is the systematic study of the well-known Aristotelian square of oppositions, and its various extensions and variants. Bitstrings have proved to be an extremely powerful tool in addressing a wide variety of issues in this area. The main aim of this paper is to provide a unified account of bitstrings in LG, and to illustrate their success by showing how they have been used not only to obtain precise answers to open questions, but also to raise interesting new questions, thus sparking new lines of research.

Bitstrings are sequences of bits (0/1) that encode formulas from a logical system (e.g. classical propositional, first-order, modal and public announcement logic) or lexical field (e.g. comparison, color terms, set inclusion, subjective quantification). For example, the bitstring encodings of the S5-formulas □p and ◊p, i.e. β(□p) and β(◊p), are resp. 1000 and 1110. The usual Aristotelian relations can straightforwardly be defined for bitstrings; e.g., two bitstrings (of length 4) b_1 and b_2 are said to be contrary iff $b_1 \wedge b_2 = 0000$ and $b_1 \vee b_2 \neq 1111$. For most cases, the bitstring mapping β -- unlike Pellissier's setting approach -- can easily be seen as assigning a *semantics* to the formulas. Each bit provides an answer to a (binary) meaningful question (this viewpoint originates in the analysis of generalized quantifiers as sets of sets). For example, in S5 the bit positions encode answers to the following questions: is *p* true in all/some but not the actual/the actual but not all/no possible worlds?

A main advantage of bitstrings is that they allow us to study logical properties of formulas φ in terms of the bitstrings β(φ) that encode them. For example, we have shown that unconnectedness (a logical relation that was introduced for independent reasons, having to do with information in logical geometry) requires bitstrings of length at least 4: if φ and ψ are unconnected, then β(φ) and β(ψ) have at least 4 bits.

Bitstrings also play a heuristic role in obtaining diagrammatic insights, e.g. (i) when exploring the rhombic dodecahedron (RDH) as an Aristotelian and as a Hasse diagram, and (ii) when devising a systematic and exhaustive classification of all types of Aristotelian hexagons. For example, by considering the Boolean closure of the bitstrings of length 4, Smessaert found in 2003 three new Jacoby-Sesmat-Blanché hexagons inside the RDH. Generalizing this

approach, we have recently written a computer program that generates all Aristotelian hexagons that can be constructed with bitstrings of arbitrary length *n*, and determines which (possibly new) type they belong to.
Finally, bitstrings also help us generate new questions about the linguistic/cognitive aspects of the expressions they encode. For example, although the mathematical perspective does not distinguish between ‘linear’ bitstrings (such as 1010) and ‘nonlinear' ones (such as $1^{0}{}_{1}0$), this difference can be relevant from a linguistic/cognitive perspective. Linear bitstrings imply that all questions (all bits) about a lexical field can be situated on a single dimension (e.g. proportional quantification), whereas nonlinear ones imply that the various questions belong to fundamentally distinct dimensions (e.g. modality). This allows us to formulate empirical hypotheses, e.g. concerning the cognitive complexity (processing times) of these lexical fields.

Harald Stamm

The square of opposition of topologized Aristotelian quantifiers

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With the help of lattice-theoretical and topological notions new Aristotelian quantifiers are defined which give rise to a new syllogistics that could cover natural language arguments more adequately and can also be applied to the question of theory change. We define not only a typicality operator but also Aristotelian quantifiers with an explicitly epistemological motivation. Interesting formal properties of the new quantifiers will be discussed. The classical square of opposition turns out to be a special case of the new one, diagrammatically the old one lies in the new square of oppositions.
We start with a set of concepts C together with an operation & to define an abstract concept system <C,&> which is a semilattice. The extensional interpretation <i, C, U> of <C,&> is given by a set U and a semilattice map i: <C,&> –> <Pot(U), ∩ >. The set of interpreted subsets of U is a classification of U. Aristotelian syllogistics can be considered as a logic of classificatory arguments and a concept system can be considered a scientific theory and its natural logic is given by Aristotelian syllogistics at least with respect to the conceptual level. Given an extensional interpretation of an abstract concept lattice the closure operator cl is defined by: cl(V): = ∩ {i(w): w ∈ C and V c i(w)}.
An epistemological interpretation for cl is given. If cl(V) ≠ V, then cl(V) can be considered to be an Aristotelian approximation of V, i.e. one that is defined by necessary and jointly sufficient conditions. Looked at from below cl(V) V can be regarded as a kernel or prototype that generates cl(V). With <U,cl> we have a closure structure and we define an Aristotelian concept lattice <C,∧,v>, where ∧ is the conjunction or intersection of (extensionally interpreted) concepts and c1 v c2 := cl(c1 ᴗ c2). The value of v gives us something like the smallest concept or natural kind that comprises c1 and c2. With the help of cl a kernel or typicality operator is defined by: t(X):= cl(C(cl(C(X))), where “C” denotes the operation of set theoretical complement.
The classical Aristotelian logic is easily defined by using the semilattice <C, ∧>. The new syllogistics comes from defining structural quantifiers such as E´XY := {(X,Y): X v Y = 1}, where “1” denotes the trivial concept, i.e. there is no non-trivial concept that comprises X and Y in

the concept lattice in question. The classical cousin is EXY := {(X,Y): X ∧ Y = 0 }, where"0"= denotes the absurd concept.
Given these structural Aristotelian quantifiers we get arguments that are valid in one concept lattice and invalid in another. Such context-dependent syllogisms can be applied to model non-monotonic reasoning in the case of theory change. We topologize the classical Aristotelian quantifiers by applying the typicality operator to their arguments, e.g. A tX Y, meaning something like "All typical X are Y", getting lots of new syllogisms.

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Constantin Teleanu

La dialectique combinatoire des termes opposés dans la « *figura contrariorum X* » de l'Art quaternaire de Raymond Lulle

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La chronologie des variantes de l'Art de Raymond Lulle (~1232-1316) –déduite du catalogue conçu par A. Bonner– divise la refonte successive de son Art en quatre phases, mais la plupart des investigateurs de l'Art de Lulle admettent qu'il y a deux grandes étapes de l'Art de Lulle entre lesquelles Lulle établit une convenance logique. Le renfort logique du quadrangle des termes contraires aide souvent Lulle afin de résoudre diverses questions théologiques. Il s'applique aussi bien aux arts libéraux du *quadrivium* –astronomie, géométrie– qui sont renouvelés par Lulle afin de déduire finalement la quadrature du cercle. Mais Lulle n'use pas seulement du quadrangle logique des propositions contraires. Il invente encore quelques variétés du quadrangle des contraires. Le propos de notre investigation est celui de comprendre quelles applications du quadrangle des contraires sont inventées par Lulle –parfois contre Aristote– pour déduire dialectiquement toute solution requise au dénouement syllogistique des questions disputées.
La compilation du *Compendium Logicae Algazelis* de 1271-1272 aborde brièvement la figure des propositions opposées –acquise par Lulle au moyen de la *Logica Algazelis*– entre lesquelles Lulle distingue tant de concordances que de contrariétés. Il faut enjoindre la grande encyclopédie du premier Art de Lulle –donc son Art contemplatif du *Libre de contemplació en Déu* de 1273-1274– à l'étape de l'Art qui devance son abréviation inventive initiale. Le début de la distinction D^{38} du *Libre de contemplació en Déu* dessine la figure des contraires comme arbre de prédestination qui se compose de modalités opposées : *in causalitate*, *in casualitate*, *in possibilitate*, *in impossibilitate*. Il y a quelques années que M. M. Romano investiguait la figure de l'arbre des modalités contraires qui régissent la prédestination, dont M. M. Romano déduit que Lulle s'y sert bien de la figure des contraires, pour démontrer que la prédestination n'est pas contraire aux choix du libre arbitre. Ainsi Lulle résout-il une épineuse question théologique par son usage dialectique du quadrangle des contraires.
La phase quaternaire –étendue de l'*Ars compendiosa inveniendi veritatem* de 1273-1274 jusqu'au *Compendium seu commentum Artis demonstrativae* de 1289 qui marque la fin du

cycle de l'*Ars demonstrativa* de 1283– développe la composition de quelques variantes de l'Art de Lulle dont la figure A des principes absolus dépend du nombre quadrangulaire. Le montage quaternaire de l'Art de Lulle suppose donc la figure du quadrangle. Le quadrangle des contraires devient une figure constitutive de quelques diagrammes des variantes de l'Art quaternaire. Le contrecoup majeur des investigations de F. A. Yates et R. Pring-Mill suggérait que la figure du quadrangle des éléments était fondamentale pour comprendre la phase quaternaire de l'Art de Lulle, mais Lulle montre bien que son approche des qualités contraires du quadrangle élémental n'était qu'une application particulière du quadrangle des contraires. Il s'applique tant à la physique qu'à la médecine des éléments qui investigue les concordances et les contrariétés de leurs qualités propres ou appropriées. Le quadrangle des contraires de l'Art quaternaire s'applique ensuite à l'ensemble des arts.

Fabio Elias Verdiani Tfouni

Alethic and Deontic Modalities in Language and Psychoanalisys.

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This work consists on a treatment of interdiction and silence within the fields of Psychoanalysis and Discourse Analysis. This task will take place with a use of the Aristotelian logic (both alethic and deontic). Alethic: necessary, possible, impossible, and contingent or not possible. Deontic: obligatory, prohibited, permitted, and facultative. We propose that categories like silence, interdiction, prohibition, prohibition of incest, censorship, and Law can be approached by Modal Logic. We specifically argue that these categories from Discourse analysis and Psychoanalysis have alethic and deontic aspects, and that the deontic is a manifestation of the alethic. We state that it is a logical necessity of language to hold within itself the impossible. Because if there was an utterance that says everything, then there will be no place for further use of language. In other words: language has to be incomplete for utterances to take place. Therefore, there cannot be an utterance that corresponds to the universal affirmative: "all is said". How does language become incomplete? With the operation of the Interdiction. Interdiction is an operator that cuts *all* and reduces it to *some*, so that "all is said" becomes impossible, and makes it possible to say something. Interdiction also is a term to refer both to the impossible and the prohibition, since the prohibition is a local manifestation of the impossible. As Magno points: "The incest is impossible, that is why it is prohibited, I said a thousand times" (MAGNO, 1986, p.16 our translation). Some of our cultural laws are a deontic manifestation of a greater alethic law. That is the case of incest prohibition and also, what explains the logical conditions for the existence of language. All societies have censorship in the sense that all of them are regulated by the impossible of alethic laws that appears to society in the form of prohibitions. In the case of language, the impossible within its field means that some thing has to remain unsaid. In other words, something remains necessarily in silence, and this silence is not negative, it is the operator that makes it possible to say something exactly by making it impossible to say it all (everything is said). For us, Freud's examples point out that prohibition of incest is universal. This means it is impossible. We can see that the impossible of incest manifests itself by deontic prohibitions in cultures around the world. Therefore, we argue with Magno that this alethic universal law manifests itself as deontic prohibitions.

Freud's examples are very interesting for language study because the ritual involving the prohibition of incest come hand in hand with prohibitions in utterances.

Joseph Vidal-Rosset

The exact intuitionistic meaning of the square of opposition

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The exact definition of the intuitionistic square of opposition is given in this talk with help of two provers for intuitionistic logic: IMOGEN, written in Standard ML by Sean Mc Laughlin [2], and ileanCoP, a prover in Prolog written by Jens Otten [4]. Therefore, thanks to this couple of provers, I show that intuitionistic logic preserves the square of opposition as square, contrary to Mélès claims [3]. But if it is refutable that intuitionistic logic transforms the square of opposition into another geometrical figure, it is of course provable that intuitionistic logic does not preserve *all* logical relations that are classically deducible *via* the traditional square of opposition. I try to explain both the reasons and the significance of these amputations.

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Knowledge *versus* Ignorance

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In the process of cognizing or discovering reality, we acquire and gather knowledge about its objects. We live, at the same time, inside a community and culture, taking over or sharing with its members the knowledge about objects that is of interest to us. The knowledge about a given object, which is accessible to the members of a given sociocultural community, can vary greatly; it often depends on the intellectual output of a scientific or professional group, who – to a greater or lesser extent – make their knowledge available to a given agent, e.g., in the process of agent's learning or assimilating the already accumulated knowledge.

Assuming that the whole knowledge (gathered as the intellectual output of a certain scientific or professional group) about object *o* of a given fragment of reality is given and that it is represented by means of its propositional component, one can distinguish 4 types of sentences informing about the knowledge of agent *a*, in relation to the complete knowledge. Therefore:

(1) K_a All – Agent *a* knows all about object *o*
(Agent *a* knows everything about object *o*),

(2) K_a No – Agent *a* knows nothing about object *o*
(Agent *a* does not know anything about object *o*),

(3) K_a Part – Agent *a* knows partially about object *o*
(Agent *a* knows something about object *o*),

(4) K_a PartNo – Agent *a* does not know everything about object *o*
(Agent *a* does not know something about object *o*).

The four distinguished types of sentences about the agent's knowledge and ignorance form an epistemic square of opposition. They can be interpreted as applied categorical propositions of Aristotle's square of opposition.

Paul Weingartner

The Square of Opposition Interpreted with a Decidable Modal Logic

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In his short article "De Propositionibus Modalibus" Thomas Aquinas interpreted the square of opposition with the help of modalities. He observed that all the essential relations of the square: contradictions in the diagonal, contraries, subcontraries and subalternities are prevented if one puts the following modalities into the four corners: necessary, necessary-not, possible, possible-not.

In this paper it will be shown that the 24 syllogistic modes can be interpreted in this way by a decidable modal logic in such a way that those 15 which do not make existential presuppositions are valid in this modal logic and the remaining ones are valid if one adds the premise that the antecedence is possible. This modal logic is based on the 6-valued propositional logic RMQ which has relevance properties and was constructed in order to avoid paradoxes which come up if two-valued classical propositional logic is applied outside logic and mathematics, i.e. to empirical sciences (Weingartner, 2009).

Suppose that subject-term, middle-term and predicate-term are represented by the propositional variables *p, q* and *r*, then the modal interpretations of the syllogistic modes BARBARA (1st figure), FESTINO (2nd figure) and BOCARDO (3rd figure) are the following (where "*L*" means *necessary* and "*M*" means *possible*):

$L(q \to r) \wedge L(p \to q) \to L(p \to r)$ BARBARA
$L(r \to \neg q) \wedge M(p \wedge q) \to M(p \wedge \neg r)$ FESTINO
$M(q \wedge \neg r) \wedge L(q \to p) \to M(p \wedge \neg r)$ BOCARDO

For the remaining modes the possibility of the respective antecedence has to be added. Thus for example DARAPTI can be proved to be strictly valid in RMQ in the following form:

$L(q \to r) \wedge L(q \to p) \wedge Mq \to M(p \wedge r)$

This holds also for the other 8 of the 9 syllogisms which make existential presuppositions (with *Mp* or *Mr* as additional premises).
For the square of opposition interpreted in RMQ it holds: (1) the contradictions in the diagonal hold; (2) the subcontraries of the I- and O-sentence hold; (3) the contraries of the A- and E-sentence and the subaltern relations hold under the condition of satisfying the antecedence, i.e. when adding the premise that the antecedence is possible.

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Urszula Wybraniec-Skardowska

Logical Squares for Classical Logic Sentences

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Among 16 binary classical sentence-forming connectives there are only 4 for which sentences that are built by means of them are true only in one case. They are the following:

both …and…, …unless…, not…because…, nether …nor… (binegation $\downarrow$).

We define them by means of the connectives: conjunction $\wedge$ (or implication $\to$) and negation $\sim$ as follows:

both p and q $=_{df}$ $p \wedge q$;

p unless q $=_{df}$ $\sim (p \to q) \equiv p \wedge \sim q$;

not p because q $=_{df}$ $\sim (q \to p) \equiv \sim p \wedge q$;

neither p nor q $(p \downarrow q) =_{df} \sim p \wedge \sim q$.

The above conjunctive sentences (conjunctions) are pairwise *contraries*, i.e., the denial alternatives of two sentences of each of the 6 pairs of the above conjunctions are true, so the sentences can never both be true, but can both be false.

To each of the 6 pairs of *contrary* conjunctions from the following:

(i) $p \wedge q$, $p \wedge \sim q$, $\sim p \wedge q$, $\sim p \wedge \sim q$

there exist a pair of *contradictory* implications from the following pairwise *subcontrary* implications:

(ii) $p \to q$, $p \to \sim q$, $\sim p \to q$, $\sim p \to \sim q$

or a pair of *contradictory* disjunctions from the following pairwise *subcontrary* disjunctions:

(iii) $p \vee q$, $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee \sim q$

or a pair of *contradictory* denial alternatives from the following pairwise *subcontrary* denial alternatives:

(iv) p / q , p / ~ q , ~ p / q , ~ p / ~ q .

Each pair of (ii) (resp. (iii), (iv)), together with the suitable pair of conjunctions of (i), creates one of the 6 squares of oppositions for complex sentences of classical logic. As it turns out, one of them was already known to Petrus Hispanus, Pope John XXI, in the 13th century.

It is possible to consider a few squares of oppositions for sentences built from other binary classical connectives selected from all 16 possible ones.

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From logical square through logical rectangle to logical pie

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In mutually-inversistic logic constructed by the author, $P=^{-1}Q$ denotes that P is a sufficient and necessary condition of Q, $P<^{-1}Q$ denotes that P is a sufficient but not necessary condition of Q, $P\leq^{-1}Q$ denotes that P is a sufficient condition of Q, $P\neq^{-1}Q$ denotes that P is not a sufficient and necessary condition of Q. We have the well-known logical square shown in Fig. 1, which contains 6 opposition relations: the subalternation of $P<^{-1}Q$ and $P\leq^{-1}Q$, the subalternation of $Q<^{-1}P$ and $Q\leq^{-1}P$, the contrariety of $P<^{-1}Q$ and $Q<^{-1}P$, the subcontrariety of $P\leq^{-1}Q$ and $Q\leq^{-1}P$, the contradiction of $P<^{-1}Q$ and $Q\leq^{-1}P$, the contradiction of $Q<^{-1}P$ and $P\leq^{-1}Q$. The author proposes a logical rectangle shown in Fig. 2, which, in addition to the 6 opposition relations the logical square contains, contains 4 more opposition relations: the contrariety of $P=^{-1}Q$ and $P<^{-1}Q$, the contrariety of $P=^{-1}Q$ and $Q<^{-1}P$, the subalternation of $P=^{-1}Q$ and $P\leq^{-1}Q$, the subalternation of $P=^{-1}Q$ and $Q\leq^{-1}P$. The left compartment and the right compartment of Fig. 2 can be connected to form a logical pie shown in Fig. 3, which, in addition to the 10 opposition relations the logical rectangle contains, contains 5 more opposition relations: the subalternation of $P<^{-1}Q$ and $P\neq^{-1}Q$, the subalternation of $Q<^{-1}P$ and $P\neq^{-1}Q$, the contradiction of $P=^{-1}Q$ and $P\neq^{-1}Q$, the subcontrariety of $P\leq^{-1}Q$ and $P\neq^{-1}Q$, the subcontrariety of $Q\leq^{-1}P$ and $P\neq^{-1}Q$.

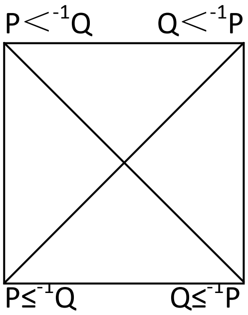

Fig.1 Logical square

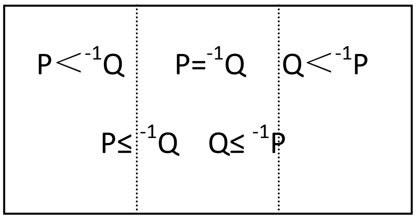

Fig.2 Logical rectangle

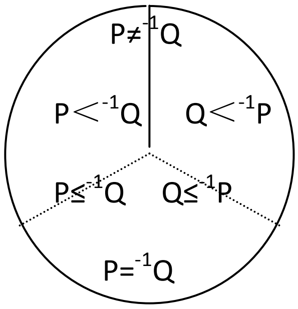

Fig.3 Logical pie

Reference

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Richard Zuber

Anaphors and the Square of Oppositions

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Anaphors (anaphoric expressions) are expressions which are syntactically like noun phrases since they can be arguments of transitive verb phrases but their referential meaning depends on the meaning of an other noun phrase. For instance the reflexive pronoun *herself* and the reciprocal pronoun *each other* are anaphoric expressions. Formally anaphors do not denote type <1> quantifiers, as do noun phrases. The pronoun *herself* denotes, roughly, the function **SELF** which applies to binary relations and gives a set of individuals as output: **SELF(R)=**{x: <x,x> in **R**}. One observes that this function is self-dual, that is **SELF(R)**=**SELF'(R')**. The denotation of *each other* is not self-dual but one can define for it two negations giving rise to the contradiction and the contrariety. It is thus possible to analyse traditional square of oppositions in the context of anaphors and this is one purpose of this paper.

I will, however, consider in addition various syntactically complex anaphors, in particular Boolean compounds of *himself/herself/themselves* as in (1) and (2), modifications of reflexive pronouns as in (3), and anaphoric determiners which can be used to form nominal anaphors, as in (4) and (5):

(1) Leo hates himself and most philosophers.

(2) Leo and Lea hate each other and some philosophers.

(3) Leo and Lea hate only themselves.

(4) Leo hates every philosopher except himself.

(5) Leo and Lea hate no philosopher except each other.

Functions denoted by such complex anaphors satisfy some general invariance principles (like *predicate invariance* or *higher order predicate invariance*) which are also satisfied by syntactically simple anaphors. These principles allow us to specify various oppositions to which these functions give rise.

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5. Index

Notes